Economic Inequality and Poverty
Economic Inequality and Poverty
International Perspectives

Editor
LARS OSBERG
## Contents

<table>
<thead>
<tr>
<th>Figures</th>
<th>vi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tables</td>
<td>vii</td>
</tr>
<tr>
<td>Introduction</td>
<td>xi</td>
</tr>
<tr>
<td>Lars Osberg</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1. The Measurement of Income Inequality</td>
<td>3</td>
</tr>
<tr>
<td>Stephen Jenkins</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Cross-National Comparisons of Inequality and Poverty Position</td>
<td>39</td>
</tr>
<tr>
<td>Timothy M. Smeeding</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Global Economic Inequality and Its Trends Since 1950</td>
<td>60</td>
</tr>
<tr>
<td>Albert Berry, François Bourguignon, and Christian Morrisson</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4. The Distribution of Household Wealth: Methodological Issues, Time Trends, and Cross-Sectional Comparisons</td>
<td>92</td>
</tr>
<tr>
<td>Edward N. Wolff</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5. The Definition and Measurement of Poverty</td>
<td>134</td>
</tr>
<tr>
<td>Aldi J. M. Hagenaars</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Patricia Ruggles</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Social Security and Income Redistribution</td>
<td>193</td>
</tr>
<tr>
<td>Paul R. Cullinan</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>8. The Impact of Government Tax and Expenditure Programs on the Distribution of Income in the United States</td>
<td>220</td>
</tr>
<tr>
<td>Patricia Ruggles</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>246</td>
</tr>
<tr>
<td>Lars Osberg</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Index</td>
<td>251</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Contributors</td>
<td>258</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.1</td>
<td>Headings in Annual Economic Statement for Arthur Dent</td>
</tr>
<tr>
<td>1.2</td>
<td>Histogram</td>
</tr>
<tr>
<td>1.3</td>
<td>Pen's Parade and the Cumulative Distribution Function</td>
</tr>
<tr>
<td>1.4</td>
<td>Histogram for Log</td>
</tr>
<tr>
<td>1.5</td>
<td>The Lorenz Curve</td>
</tr>
<tr>
<td>1.6</td>
<td>Graphic Representation of SWF for n = 2</td>
</tr>
<tr>
<td>1.7</td>
<td>The Welfare Weight Function and Transfer Sensitivity</td>
</tr>
<tr>
<td>1.8</td>
<td>Alternative Distance Functions, h(s)</td>
</tr>
<tr>
<td>3.1</td>
<td>The Evolution of Economic Inequality</td>
</tr>
<tr>
<td>4.1</td>
<td>Share of Total Wealth Held by the Top 1 Percent 1920–81 (based on traditional wealth measures)</td>
</tr>
<tr>
<td>4.2</td>
<td>Share of Total Wealth Held by the Top 1 Percent, 1920–81 (including government and private pension wealth)</td>
</tr>
<tr>
<td>5.1</td>
<td>The Definition of Aggregate Poverty Indices: Head-count Ratio</td>
</tr>
<tr>
<td>5.2</td>
<td>The Definition of a Poverty Line</td>
</tr>
<tr>
<td>5.3</td>
<td>The Definition of Aggregate Poverty Indices: Average Poverty Gap</td>
</tr>
<tr>
<td>5.4</td>
<td>Poverty Index, Defined as Welfare Gap</td>
</tr>
<tr>
<td>7.1</td>
<td>Income and Consumption over the Life Cycle</td>
</tr>
<tr>
<td>7.2</td>
<td>Social Security Benefit Formula for Those Becoming Eligible for Benefits in 1990</td>
</tr>
<tr>
<td>7.3</td>
<td>Earnings Patterns of Four Birth Cohorts, 1957–69</td>
</tr>
</tbody>
</table>
Tables

1.1 Frequency Distribution Example 10
1.2 Lorenz Curve Data 14
2.1 Inequality Measures (x 1000) for Disposable Income (DPI) and Adjusted Disposable Income (ADPI) and Rank Order (RO) for Each Measure 45
2.2 Relative Economic Position of Persons Living in Various Demographic Groups 48
2.A-1 An Overview of LIS Datasets 56
3.1 Average Per-Capita Income and Distribution of Income, Selected Major Countries 62
3.2 Relative Per-Capita Expenditures of Selected Countries vis-à-vis the United States Including and Excluding Services, for Various Weighting Systems, 1970 66
3.3 Estimates of World Personal Income and Consumption Distribution in 1970 68
3.4 World Distribution of GNP and Consumption Including Socialist Countries: Selected Years 72
3.5 World Distribution of GNP and Consumption Excluding Socialist Countries: Selected Years 76
3.6 Average Annual GDP Growth during 1965–80 and 1980–85, by Regions 78
3.7 Country Composition of Selected Quantiles of the World Distribution of GNP, 1950 and 1977 80
3.A-1 Countries Included in the Analysis 84
4.1 Shares of Total Household Net Worth Held by Richest Individual Wealthholders in the United Kingdom, 1923–80 102
<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2</td>
<td>Shares of Total Household Taxable Net Worth Held by Richest Households in Sweden, 1920–83</td>
<td>107</td>
</tr>
<tr>
<td>4.3</td>
<td>Shares of Total Household Net Worth or Total Assets Held by Richest Individual Wealthholders in the United States, 1922–81</td>
<td>109</td>
</tr>
<tr>
<td>4.4</td>
<td>Inequality Measures for Different Concepts of Household Wealth, Based on Both Unadjusted and Adjusted U.S. Data, 1962 and 1983</td>
<td>113</td>
</tr>
<tr>
<td>4.5</td>
<td>Share of Total Net Worth of Richest Households in the United States: Estimates from Survey Data, 1962–84</td>
<td>115</td>
</tr>
<tr>
<td>4.6</td>
<td>A Comparison of Shares of Top Wealthholders Based on 1962 Estate Tax and Household Survey Data for the United States</td>
<td>118</td>
</tr>
<tr>
<td>4.7</td>
<td>The Concentration of Wealth in France: Estimates from Alternative Sources</td>
<td>119</td>
</tr>
<tr>
<td>4.8</td>
<td>Raw Age-Wealth (HDW) Profiles from Various Sources, 1962–83</td>
<td>122</td>
</tr>
<tr>
<td>4.9</td>
<td>Percentage Composition of Household Wealth by Income and Wealth Class, 1969</td>
<td>126</td>
</tr>
<tr>
<td>6.2</td>
<td>Percentage Distribution of Poor Households by Work Status of Head, Selected Years, 1967–84</td>
<td>166</td>
</tr>
<tr>
<td>6.3</td>
<td>Percentage of Entrants into Poverty Remaining at Selected Months, by Characteristics of Entrant</td>
<td>168</td>
</tr>
<tr>
<td>6.4</td>
<td>Civilian Employment/Population Ratio, 1979–84</td>
<td>171</td>
</tr>
<tr>
<td>6.5</td>
<td>Estimated Size of the Underclass: Review of Several Studies</td>
<td>174</td>
</tr>
<tr>
<td>6.6</td>
<td>Estimates of the Demographic Composition of the Persistently Poor Population, from Various Studies</td>
<td>183</td>
</tr>
<tr>
<td>7.1</td>
<td>Percentage of Work Force Covered, Average Retired Worker Benefit, Average Wages, Selected Years 1940–87</td>
<td>199</td>
</tr>
<tr>
<td>7.2</td>
<td>Social Security Taxes</td>
<td>201</td>
</tr>
<tr>
<td>7.3</td>
<td>Major Characteristics of Social Security Financing, Selected Years 1940–2060</td>
<td>202</td>
</tr>
<tr>
<td>7.4</td>
<td>Labor Force Participation Rates by Age, 1950–86</td>
<td>206</td>
</tr>
<tr>
<td>TABLES</td>
<td>ix</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>Average Monthly Benefits for Retirees Age 65 in 1988 with Different Earnings</td>
<td>208</td>
</tr>
<tr>
<td>7.6</td>
<td>Comparisons of Total Benefits for Couples with Different Shares of Total Earnings</td>
<td>210</td>
</tr>
<tr>
<td>7.7</td>
<td>Average Household Income by Age of Head, 1975 and 1985</td>
<td>211</td>
</tr>
<tr>
<td>7.8</td>
<td>Average Earnings for Those with Earnings in 1985, by Sex and Age</td>
<td>212</td>
</tr>
<tr>
<td>7.9</td>
<td>Antipoverty Effectiveness of Cash and Noncash Transfers for Individuals in Units with All Members 65 or Older, Selected Years</td>
<td>215</td>
</tr>
<tr>
<td>7.10</td>
<td>Social Security as a Proportion of Total Income for Age Units 65 or Older, 1986</td>
<td>216</td>
</tr>
<tr>
<td>8.1</td>
<td>Sources of Revenue for the Federal Government, Fiscal Year 1987</td>
<td>229</td>
</tr>
<tr>
<td>8.2</td>
<td>Sources of State and Local Revenues, Fiscal Year 1985–86</td>
<td>229</td>
</tr>
<tr>
<td>8.3</td>
<td>Distribution of Family Income and of Federal Tax Payments by Population Decile, 1988</td>
<td>230</td>
</tr>
<tr>
<td>8.4</td>
<td>Federal Outlays by Function, Fiscal Year 1987</td>
<td>232</td>
</tr>
<tr>
<td>8.5</td>
<td>State and Local Outlays by Function, Fiscal Year 1984–85</td>
<td>233</td>
</tr>
<tr>
<td>8.6</td>
<td>Distribution of Transfer Income by Population Decile, 1977, 1984, and 1988</td>
<td>234</td>
</tr>
<tr>
<td>8.7</td>
<td>1987 Family Incomes as Percentage of the Median for Family Type, by Selected Income Sources</td>
<td>236</td>
</tr>
<tr>
<td>8.8</td>
<td>Median 1987 Family Incomes as Proportion of the Poverty Line by Family Type and Income Source</td>
<td>240</td>
</tr>
<tr>
<td>8.9</td>
<td>Antipoverty Effectiveness of Tax and Transfer Programs, 1986</td>
<td>242</td>
</tr>
</tbody>
</table>
Economic inequality has always been one of the central issues of any social system. From the very earliest times, philosophers have debated the ethics of the coexistence of great wealth and abject poverty, while political analysts have emphasized the linkages between economic affluence and political influence. The central question of the modern discipline of sociology is the analysis of structured social inequality, which is largely derived from differences in command over economic resources. And economists have always claimed that the criteria for economic policy are efficiency and equity. But despite the importance of the issue, vast gaps in our knowledge about economic inequality remain.

In part, these gaps are due to the fact that economists cannot all agree on what the crucial questions about economic inequality really are. For some, the important issue is the differences among all individuals in potential command over goods and services—if society as a whole produces a certain amount of output, how is the total pie sliced up? For some, the situation of the poor is a particularly important dimension of inequality, since there is a long tradition of ethical concern with inequality which stresses the relative well-being of the most disadvantaged members of society. For others, the crucial questions are those which surround the concentration of ownership, wealth and power in capitalist societies. And for still others the social policies which affect inequality command the most interest.

This book of readings touches on all these dimensions of economic inequality. One of the themes running through the chapters is the crucial importance of conceptual and measurement issues to our perception of the nature of aggregate economic inequality, poverty, and the concentration of wealth and social policy. Another theme which pervades these readings is the value of international comparisons of the evidence on economic inequality. If we are to evaluate carefully the theories which seek to explain inequality and poverty or the policies which
attempt to affect it, we should have a clear idea of what exactly it is that we are seeking to explain or to influence. Our evaluation of reasonable theories or practicable policies will also depend on the range of variation which can and does exist in human societies.

At the most general level, “economic inequality” means in this book “differences among people in their command over economic resources.” By this definition we mean to distinguish between the study of actual differences among people (a factual exercise) and the study of unjustifiable differences among people (an ethical exercise). We therefore draw a distinction between the study of economic inequality and of economic inequity, while recognizing that much of the motivation for studying inequality comes from a concern with inequity. The readings in this book are mostly concerned with actual differences among people, not potential differences, and in that sense the subject of this volume is inequality of economic outcomes, rather than inequality of economic opportunity.

If we are to make comparisons, either across societies or over time, of the extent of economic inequality, we must summarize in some statistical measure (or measures) of aggregate economic inequality the differences among a large number of people in command over economic resources. Such summarization will implicitly provide a weighting of the many differences between actual individuals in society. Stephen Jenkins, in chapter 1, discusses the minimal properties which an ethically defensible measure of income inequality should possess and investigates the properties of different measures of inequality used in the economics literature. Jenkins asks: “Under what conditions can one say that inequality in one income distribution is greater (or less) than that in another income distribution? What social values are implicit in a particular measure of inequality?”

In chapter 2, Timothy M. Smeeding compares the inequality of income distribution and the incidence and distribution of poverty within ten of the world’s developed economies. As he emphasizes, some broad generalizations emerge from the data—notably the relatively high level of inequality and poverty in the United States and the low levels of both in Scandinavian countries—but for many of the most interesting policy issues we must look beneath the surface of the aggregative statistics.

Although the way in which humanity is divided into nation states is politically important, it is essentially arbitrary, the end result of a large number of historical accidents. If we have an ethical concern with economic inequality, there is no ethically defensible reason why such concern should stop at national frontiers. As Albert Berry and his collaborators demonstrate in chapter 3, differences among nations, rather than economic differences within nations, dominate global inequality. Since countries vary in their rate of economic development, the pattern of world inequality changes over time, in response to economic policies and world events. These differences between and within countries can be seen as
INTRODUCTION

"natural experiments," which illustrate the fact that aggregate economic inequality, at both the national and the global level, is something which is amenable to change.

Although most discussion of economic inequality is framed in terms of inequality of incomes, the terms "rich" and "poor" taken literally, refer to wealth, not income. In chapter 4, Edward N. Wolff asks, "What is the proper definition of 'wealth'? What difference does our definition of wealth make to our perception of inequality in wealth? What trends in wealth inequality do we observe in advanced economies?" These issues are particularly important if one believes that the possession of some forms of wealth (e.g., equity capital) entails economic power or political and social influence.

However, although a political concern with potential concentrations of wealth and power motivates some of the interest in inequality, much of it also is derived from an ethically based concern for the poor. In chapter 5, Aldi Hagenaars emphasizes that the measure of poverty which we should adopt will depend on just what we understand as the meaning of being "poor." Those who think being poor means being unable to purchase a specific bundle of necessities will argue for one definition of poverty, while those who think that poverty consists in being unable to afford a decent standard of life will have a very different perception of the nature and extent of poverty.

Even those who agree on the extent of poverty at a point in time might still disagree on the significance of poverty. Those who study poverty because of a Rawlsian concern for the well-being of the least-advantaged members of society will feel that the primary focus of poverty policy should be on those who will be poor over their lifetimes. Those whose concern with poverty stems from a commitment to equality of opportunity will want to know how much of the poverty population is part of a "culture of poverty" which blights the lives of the children it touches. And those who emphasize the social insurance role of the state in an uncertain world will want to know how many of the poor are there temporarily, and why. In chapter 6, Patricia Ruggles uses a particular definition of poverty—that of the Social Security administration of the United States—to examine the issue of the permanence of poverty, and the reasons why some of the poor can expect to be poor for most of their lives.

This distinction between poverty in one year and poverty over a lifetime is really a special case of the distinction between annual income and lifetime income which is crucial to much of the discussion of economic inequality. In chapter 7, Paul Cullinan examines a particular government program, Social Security in the United States, which redistributes income between different years of an individual's lifetime, as well as between individuals of the same generation and between different generations of individuals. As he makes clear, a whole series of distributional issues are raised by this one government program.

Finally, in chapter 8, Patricia Ruggles considers the impact of the full spectrum of government tax and expenditure activities on the distribution of income.
in the United States. As Cullinan’s chapter indicates, it is not easy to assess the impact on inequality of one program, and it is obviously much more difficult to assess the impacts of all programs considered jointly. It is especially difficult to come to a firm conclusion about the incidence of taxes such as the corporate income tax or the benefits of some types of expenditures (e.g., on defense). However, one can evaluate the impacts of personal income taxation, cash transfers and in-kind benefits to households on the rate of poverty and on the distribution of income in the United States. Transfer programs do have a significant impact on the overall poverty gap and on the poverty rate, but the net impact of government on the degree of aggregate inequality in the United States is relatively small.

In drawing these readings together, we are all very conscious that much more could be written, and should be written, on economic inequality. Nevertheless, all the readings in this volume are original articles which seek to present some examples of the state of the art of the theory and measurement of economic inequality and poverty. We hope that they will prove useful to analysts of economic inequality and that they will help to stimulate further research on this important subject.

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Economic Inequality and Poverty
1. Introduction

In Douglas Adams' science fiction comedy classic *The Hitchhiker's Guide to the Galaxy*, supercomputer Deep Thought revealed that the Answer to the Ultimate Question of Life, the Universe, and Everything was "42." But unfortunately no one knew what the Question was! Newcomers to the economic inequality literature might justifiably claim to have the opposite problem. There are well-defined important questions—for example, is inequality increasing or decreasing, and by how much?—but what on earth does it mean (if anything) when the Gini coefficient is 42 percent? The aim of this chapter is to help resolve such problems and thereby provide a theoretical framework for the empirical chapters following.

An essential preliminary to any inequality (or poverty) study is clarification of the nature of the distribution to be analyzed to ensure that it represents the appropriate concept of economic power and does so for each constituent unit. Important questions are: What is the economic variable of interest? What is the demographic unit to which this pertains, and how does one ensure variables are comparable across households? These issues are the subject of section 2.

The subsequent sections consider inequality measurement per se, focusing on two sorts of Ultimate Question. The first is: under what conditions can one say unambiguously that one distribution is more or less unequal than another? This covers issues such as whether or not inequality has increased over the last decade, is larger in the United States than in the United Kingdom, or whether the post-tax income distribution is more equal than the pretax one. A more demanding requirement is not only the
ability to rank the distributions in terms of their inequality, but also to know how much more or less unequal one is relative to the other. On this basis a distinction can be made between ordinal inequality measures (the first case), and cardinal ones (the second). The advantage of the first sort of measures is that the set of basic assumptions underpinning them is smaller than for the second sort—which increases the possibility of having society-wide agreement about their use—but the penalty is that there are situations where unambiguous rankings cannot be made. However, using measures incorporating extra assumptions (the second class) extends the range of potential conclusions that can be drawn. Note that two summary measures may rank a set of distributions in exactly the same way, but represent this using different numerical scales; in this situation the measures are said to be ordinally, but not cardinally, equivalent.

At the heart of the survey is an investigation of the properties of different measures used in the literature and their implications. Section 3 discusses the information provided by several alternative graphical summaries of distributions such as histograms, Pen's Parade, and Lorenz curves, plus the properties of some related and commonly-used measures of a distribution's dispersion, including the Gini coefficient. Inequality measures for ranking distributions are considered in section 4, where attention is drawn to the links between inequality orderings, nonintersecting Lorenz curves, and certain underlying axiomatic properties, and to analogous results in the literature on decision making under uncertainty. Section 5 focuses on two important classes of cardinal measures. The first is the Atkinson family derived from sets of assumptions about society's Social Welfare Function. The Generalized Entropy family of indices considered second has a rather different pedigree but is shown to be closely related to the Atkinson one and to have an additional property very useful for empirical work, namely, additive decomposability by population subgroup. This property can be used to answer important questions such as: how much of overall inequality can be attributed to differences in household composition, or age differences? Section 6 provides concluding comments. It reiterates the main themes of the chapter and draws attention to some other topics not covered.

It needs to be stressed that this chapter provides only an introduction to the topic of inequality measurement, and concentrates on theoretical aspects. As a survey it relies heavily on previous work. For these reasons a brief Further Reading section has been included, providing a selective annotated guide to the literature.

2. Essential Preliminaries

The Variable of Interest

The study of economic inequality is the analysis of differences across the population in access to, and control over, economic resources. Consider how the concept italicized might be measured, taking first the situation of a single work-
Figure 1.1. Headings in Annual Economic Statement for Arthur Dent

Income in year $t$  
(a) earnings  
(b) income from savings (including occupational pensions)  
(c) capital receipts (gifts, inheritances, etc.)  
(d) transfers from the government

Minus

Outgoings in year $t$  
(e) expenditures  
(f) capital transfers (gifts made, etc.)  
(g) taxes

Equals

Savings in year $t$.  
Net wealth at end of year $t = $net wealth at end of year $t-1$, plus savings in year $t$.

ing man, Arthur Dent (complications arising from having other types of demographic unit are considered below). In any given year one could draw up a personal economic statement for Dent, just as the Treasury Department does for the country, summarizing his income and expenditures during the year plus his assets and liabilities at the year’s end. See Figure 1.1.

The most commonly used variable in inequality analyses is income, but this may be defined in many ways. An emphasis on the potential economic power under Dent’s own direct control suggests the use of (i) pre-tax-and-transfer income $= (a) + (b) + (c)$. This may be contrasted with a measure based on (e) expenditure which reflects the exercise of power. This is rarely used—miserly millionaires are not usually considered poor—but on the other hand consumption of specific resources may sometimes be of special importance. A good example is food in underdeveloped countries. Dent’s overall economic power also depends on the effects of government. This might be summarized by incorporating (d) and (g) into (i) to give (ii) post-tax-and-transfer income, but there remain ambiguities.

A case could be made for excluding indirect taxes paid (for example local sales taxes), since otherwise (ii) may simply reflect differences in consumption preferences, not in the resources at Dent’s disposal. Noncash income such as the value of government-provided health insurance or housing subsidies, employment-related fringe benefits, or the value of home-produced activities could also be included. Although difficult to value, these increase Dent’s total purchasing power, and should be included in a comprehensive definition of income. More controversially, there have also been proposals to incorporate the value of an individual’s leisure time as well. If everyone is freely able to vary their hours of work, then hours spent in leisure represent foregone wages and hence potential economic power, but this amounts to assuming away the existence of involuntary unemployment and is obviously controversial.
Valuation problems are also particularly relevant to the measurement of capital receipts. A gift of $1,000 to Dent from his uncle can be handled relatively straightforwardly, but what if instead he was given an Andrew Wyeth original or a block of IBM shares. And if either of these were to appreciate in value, the resultant capital gain is an increase in potential income, realizable if the assets were to be sold. On the other hand the actual purchasing power clearly depends on the marketability of the assets.

Of course asset holdings are relevant to power over economic resources in their own right, not just for the income they yield. There is an ongoing debate about the nature of the relationship between wealth and power, and whether certain assets deserve special emphasis—historically, land ownership has been associated with influence and status in society. And the principal shareholder in a large, private company clearly has a different type of control over society’s resources than someone with the same amount of savings held in a savings account, or tied up in a pension scheme.

In summary, there is a large range of potential variables corresponding to alternative definitions of economic power and one’s choice will depend on the particular purpose at hand. In practice one is often also constrained by what is included in the statistics available. This must be taken account of when drawing conclusions.

**The Time Period**

The cautionary remarks just made apply with equal force to the question of the appropriate time period over which income should be measured. Should it be a month, a year (as above), or perhaps a lifetime? Each gives different answers: lengthening the time period will tend to reduce the degree of dispersion observed across the population, because averaging over time tends to iron out fluctuations, producing income differences between people which are smaller than in any given time period.

Which interval length is of most social relevance will depend on how much Dent’s income actually fluctuates over time, and on his ability to transfer income between time periods if desired by borrowing or lending. For workers with substantial commission or piece-rate elements to their earnings, incomes may fluctuate dramatically from month to month. This may be of no social concern if rich brokers and poor bricklayers can borrow and lend on equal terms, but generally they cannot. The rich are more likely to have assets that may be liquidated to tide over pressing emergencies or, as collateral, help secure cheaper borrowing. Nevertheless with incomes tending to even themselves out over several pay periods the primary social interest is probably whether the fluctuations lead to incomes falling below certain minima, which would reflect a concern with poverty rather than inequality.

Systematic variations in income over individuals’ lifetimes are of greater
significance. As a young medical intern Arthur Dent may this year earn less than a similarly aged car assembly worker, but the knowledge that Dent's subsequent career earnings are likely to be much greater substantially moderates any assessment of the current situation as representing "genuine" inequality. Two main strategies are open to researchers. First, "lifetime income"—the total net present value of the incomes in different periods—may be calculated, but this is rarely done because of a lack of appropriate longitudinal data sets. The remarks above about access to capital markets apply here too. An alternative approach is to isolate the contribution of age differences to overall inequality using decomposition methods; see section 1.5.

The Income Unit

The next set of issues concerns the demographic unit used in the analysis. Arthur Dent may have two-thirds the income of Patricia McMillan, but perceptions of the inequity of this depend on whether each is single, or has a spouse and children to support and hence greater needs. The most important distinctions to be made are between individuals, families, and households, where the last also includes individuals at the same address who are not part of the nuclear family (such as grandparents or unrelated lodgers).

Although it is the well-being of individuals that is of ultimate interest, using the observed incomes of each family member is likely to be a poor proxy. Although some people, notably children, have no income in their own name, they benefit from income sharing within the household or family in which they live. The terms "family" and "household" are used interchangeably in the rest of this chapter, but it should be remembered that they are not substitutes in practice. With secular changes in demographic structure in the United States, trends in measured inequality are sensitive to the definition chosen.

The simplest adjustment to observed incomes that we could make would be to assume equal sharing of income among all members and to calculate incomes per capita, but this is unsatisfactory; equal per capita incomes do not necessarily reflect equal commands over economic resources. Unfortunately there is little information available about patterns of income redistribution within families and in particular on the extent to which breadwinners (typically men) transfer resources to those with the major domestic responsibilities (typically women).

One adjustment to the data is based on the view that the appropriate income deflator is not the actual numbers of individuals per family but the numbers of "equivalent adults." All households require items such as light and heating but since additional members can also benefit at little extra cost from their provision, there are economies of scale in larger units. However, children and adults do not have equal needs; $100 is likely to go further in a two-adult one-child family than in a three-adult one.
Equivalence Scales

An index which deflates family incomes by a score that may be less than one for each extra member is what is known as an "equivalence scale." "Equivalent income" for a household equals observed income divided by the equivalence scale value for households of its type. Not surprisingly there is considerable and continuing debate about the derivation of such scales. If there were a social consensus about the minimum necessary consumption requirements (food, clothing, housing standards, etc.) for families of different types, then one could straightforwardly specify these, cost them using assumptions about prices, and hence derive the income levels just sufficient to meet total expenditure needs. The problem is that no such agreement exists, about even dietary requirements let alone other items and, typically, not everyone can buy the items at the same set of prices.

An alternative approach is to derive the scales from data sets with information on actual household incomes and expenditure patterns. As the share of a family's budget spent on necessities such as food tends to decline with income for all household types, this budget share may be used as an indicator of household well-being. Having estimated the relationship between food share and income from the data, the differences in total income across households at a particular food share level can provide measures of the differences in income required to reach the same level of welfare; the equivalence scales required. The approach can be further developed to incorporate more sophisticated relationships between household utility and expenditure patterns (not just food shares), drawing on advanced econometric methods.

However, the use of advanced techniques like these should not blind one to the essentially normative aspects of the problem—for example, the prior choice of the characteristics used to distinguish family types. The number of adults and children (and sometimes their age) are the most commonly used, but a case can be made for also distinguishing differences in expenditures between the disabled (or the elderly) and others, or allowing for differences in labor force participation.

The equivalence scale implicit in the United States official poverty line incorporates elements of both the minimum consumption and expenditure approaches. For families of different sizes a Department of Agriculture dietary plan is used to specify the minimum necessary food requirements and the overall poverty line is taken as a multiple of this—chosen to ensure that the budget share for food is not atypically large—a judgment made on the basis of expenditure survey data. (The measurement of poverty is discussed in more detail in the chapter by Aldi Hagenaars.)

The Weighting of Income Units

The final task in constructing a distribution for analysis is to indicate the number of income units with a given income. Suppose household $i$ has income $y_i$, num-
ber of members $n_i$, and number of equivalent adults $r_i$. Even if it is agreed that $i$’s equivalent income $y_i/r_i$ is the best estimate of resources received per member, there remains the choice of whether the family should count as one, $n_i$, or $r_i$ units. As incomes are measured per head then the most consistent position is to use either of the last two as the weights. Which of these should be preferred is related to the philosophical issue of whether one should regard all individuals as counting equally within the population in question, or whether children for example should not be rated as much. Note that both strategies may provide a distribution with a shape, and hence measured inequality, which is different from that produced using data on incomes weighted per household.

3. Graphical Representations and Some Common Inequality Measures

From now on it will be assumed that all the essential preliminaries have received their due attention, and there is now a set of comparable distributions. For convenience the variable of interest will be referred to as “income” and the demographic unit as the “individual.” Much inequality analysis builds on foundations provided by statisticians, who are long accustomed to summarizing distributions. This section discusses several graphical representations of the basic data, and then some commonly-used inequality measures related to these. An analysis of their properties provides the motivation for the measures considered in later sections.

**Graphical Representations**

The most common graphical device is probably the histogram, which is based on the frequency distribution and shows the numbers of individuals with income in specified ranges. Example data are given in Table 1.1 and Figure 1.2 is based on this, with the diagram drawn so that equal areas on the chart represent equal relative frequencies. The general shape—skewed to the right, with the mean above the median—is typical. One problem immediately apparent is that the figure does not include all the information in the table: in order to picture the lower middle and bottom of the distribution, very large incomes have had to be omitted.

An alternative method and one that highlights the presence of very large incomes is the so-called Pen’s Parade. Suppose everyone in the population is represented by a person who has a height proportional to his income. Now line these agents up in order of height with the tallest at the front and have them all march past a certain spot in an hour. At a distance the silhouette of the parade would be as in Figure 1.3(a). (The horizontal axis has been scaled by expressing people’s positions as cumulative population shares. Allocating each individual in order a number from 1 to $n$—$n$ is the population size and is the number given to the tallest person—these shares equal $i/n$ for each $i = 1, \ldots, n$.) Heights increase
Table 1.1

**Frequency Distribution Example**

<table>
<thead>
<tr>
<th>Income (£ 000s)</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Cumulative percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0–0.5</td>
<td>6</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0.5–1.0</td>
<td>13</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>1.0–1.5</td>
<td>58</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>1.5–2.0</td>
<td>375</td>
<td>5.0</td>
<td>6.0</td>
</tr>
<tr>
<td>2.0–2.5</td>
<td>507</td>
<td>6.7</td>
<td>12.8</td>
</tr>
<tr>
<td>2.5–3.0</td>
<td>433</td>
<td>5.8</td>
<td>18.5</td>
</tr>
<tr>
<td>3.0–3.5</td>
<td>433</td>
<td>5.8</td>
<td>24.3</td>
</tr>
<tr>
<td>3.5–4.0</td>
<td>396</td>
<td>5.3</td>
<td>29.5</td>
</tr>
<tr>
<td>4.0–4.5</td>
<td>379</td>
<td>5.0</td>
<td>34.6</td>
</tr>
<tr>
<td>4.5–5.0</td>
<td>390</td>
<td>5.2</td>
<td>39.8</td>
</tr>
<tr>
<td>5.0–5.5</td>
<td>389</td>
<td>5.2</td>
<td>44.9</td>
</tr>
<tr>
<td>5.5–6.0</td>
<td>394</td>
<td>5.2</td>
<td>50.2</td>
</tr>
<tr>
<td>6.0–6.5</td>
<td>425</td>
<td>5.7</td>
<td>55.8</td>
</tr>
<tr>
<td>6.5–7.0</td>
<td>369</td>
<td>4.9</td>
<td>60.7</td>
</tr>
<tr>
<td>7.0–7.5</td>
<td>367</td>
<td>4.9</td>
<td>65.6</td>
</tr>
<tr>
<td>7.5–8.0</td>
<td>301</td>
<td>4.9</td>
<td>69.6</td>
</tr>
<tr>
<td>8.0–9.0</td>
<td>525</td>
<td>7.0</td>
<td>76.6</td>
</tr>
<tr>
<td>9.0–10.0</td>
<td>466</td>
<td>6.2</td>
<td>82.8</td>
</tr>
<tr>
<td>10.0–11.0</td>
<td>353</td>
<td>4.7</td>
<td>87.5</td>
</tr>
<tr>
<td>11.0–12.0</td>
<td>237</td>
<td>3.2</td>
<td>90.6</td>
</tr>
<tr>
<td>12.0–13.0</td>
<td>189</td>
<td>2.5</td>
<td>93.2</td>
</tr>
<tr>
<td>13.0–14.0</td>
<td>130</td>
<td>1.7</td>
<td>94.9</td>
</tr>
<tr>
<td>14.0–15.0</td>
<td>86</td>
<td>1.1</td>
<td>96.0</td>
</tr>
<tr>
<td>15.0–16.0</td>
<td>72</td>
<td>1.0</td>
<td>97.0</td>
</tr>
<tr>
<td>16.0–17.0</td>
<td>51</td>
<td>0.7</td>
<td>97.7</td>
</tr>
<tr>
<td>17.0–18.0</td>
<td>35</td>
<td>0.5</td>
<td>98.1</td>
</tr>
<tr>
<td>18.0–19.0</td>
<td>28</td>
<td>0.4</td>
<td>98.5</td>
</tr>
<tr>
<td>19.0–20.0</td>
<td>26</td>
<td>0.3</td>
<td>98.8</td>
</tr>
<tr>
<td>20.0–25.0</td>
<td>54</td>
<td>0.7</td>
<td>99.6</td>
</tr>
<tr>
<td>25.0–30.0</td>
<td>20</td>
<td>0.3</td>
<td>99.9</td>
</tr>
<tr>
<td>30.0–</td>
<td>13</td>
<td>0.2</td>
<td>100.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7520</td>
<td>100.0</td>
<td></td>
</tr>
</tbody>
</table>

*Source:* Author's calculations from the microdata tape for the 1981 UK Family Expenditure Survey. Income is *net household disposable income per annum* = total household money income including cash transfers, less direct taxes. The variable is unadjusted for differences in family size, etc., and each household receives a weight of one. (Five households, with zero or negative incomes, are excluded from the analyses.) Mean income is £6729 and median is £5981.
slowly for a very long time and it is over half an hour before the representative with mean income ($\mu = \frac{1}{n}\sum y_i$) passes the finishing post, and the one with twice mean income only arrives in the last five minutes. But there is then a dramatic change in heights; indeed to include the largest income in the sample used, the page would have to be extended vertically by over two yards. And if the late Paul Getty had sent a representative, his height would be several miles high! There is a close relationship between this parade and the frequency distribution; using the latter, calculate for each possible income level the total number of individuals having incomes below this (see Table 1.1, column 2). Dividing these cumulative frequencies by $n$ and plotting the results against income yields Figure 1.3(b), a graph with exactly the same shape as Figure 1.3(a).

By taking a logarithmic transformation of income it is possible to include very rich people on the graph as well. In this case, shown in Figure 1.4, equal distances along the horizontal axis now correspond to equal proportionate, not absolute, differences in income so high incomes are now compressed.

The final graphical device to be considered, and one which turns out to have particular relevance for subsequent sections, is the Lorenz Curve (LC). Call again upon the representatives used in Pen's Parade, ensuring they are still ranked the same, but this time make each agent's height equal to the sum of the individual shares in total income held by him and all the people behind (i.e., the same height or shorter). The new parade's silhouette is that pictured in Figure 1.5; more formally it is the graph of cumulative income shares.
against cumulative population shares, defined above. Example data are given in Table 1.2. Because of the way people are ranked, heights must range from zero to one and the graph be convex towards the bottom right hand corner. Note that if everyone had the same income the Lorenz Curve would lie along the diagonal line, and if all income were held by just one person the curve would lie along the axes. The two parades are closely related since the slope of the Lorenz Curve at any given cumulative population share is in fact equal to $y_i/\mu$, which is directly proportional to the height of Pen's Parade ($y_i$).

It should now be apparent that there is no obvious best way of representing income distributions graphically; each uses the same basic data but gives a different perspective. However, as far as inequality measurement is concerned, it will be seen below that the Lorenz Curve has some very important properties.
A first measure of inequality, suggested by Figures 1.1 and 1.2, is the range, the difference between the largest and smallest incomes. An immediate difficulty with this is that if every income is increased by the same proportion inequality would rise, even though the distribution retains the same fundamental shape. This may be dealt with by deflating all incomes by the mean, thence giving a mean-independent (alternatively, scale invariant, or relative) measure. Nevertheless even if adjusted, the range would still completely ignore information about income differences between the extreme values, and a distribution which had an even spread of people between these points would be given exactly the same inequality score as one within which everyone was concentrated at either one
point or the other. Indeed any measure based on distributional information at just one or two cut-off points (for example the "income share of the top 5 percent," or the "ratio of mean income among the richest 10 percent to that among the poorest 10 percent") will have the undesirable property of being completely insensitive to transfers of income over large sections of the distribution. Turn then to measures using data on all incomes.

The most commonly used measure in statistics of dispersion within distributions is the variance $V$ (or its square root, the standard deviation):

$$V = \frac{1}{n} \sum_{i=1}^{n} (y_i - \mu)^2$$

This is not mean-independent, but standardizing using the mean, $\mu$, yields the coefficient of variation,

$$C = \sqrt{V} / \mu,$$

or its square, $C^2 = V / \mu^2$ (which is ordinarily, though not cardinally, equivalent). Using log income analogous measures can be based on the formula

<table>
<thead>
<tr>
<th>Cumulative share of population (%)</th>
<th>Cumulative share of total income (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>2.8</td>
</tr>
<tr>
<td>15</td>
<td>4.6</td>
</tr>
<tr>
<td>20</td>
<td>6.8</td>
</tr>
<tr>
<td>25</td>
<td>9.3</td>
</tr>
<tr>
<td>30</td>
<td>12.1</td>
</tr>
<tr>
<td>35</td>
<td>15.3</td>
</tr>
<tr>
<td>40</td>
<td>18.9</td>
</tr>
<tr>
<td>45</td>
<td>22.8</td>
</tr>
<tr>
<td>50</td>
<td>27.0</td>
</tr>
<tr>
<td>55</td>
<td>31.6</td>
</tr>
<tr>
<td>60</td>
<td>36.6</td>
</tr>
<tr>
<td>65</td>
<td>41.9</td>
</tr>
<tr>
<td>70</td>
<td>47.7</td>
</tr>
<tr>
<td>75</td>
<td>53.9</td>
</tr>
<tr>
<td>80</td>
<td>60.7</td>
</tr>
<tr>
<td>85</td>
<td>68.1</td>
</tr>
<tr>
<td>90</td>
<td>76.3</td>
</tr>
<tr>
<td>95</td>
<td>85.8</td>
</tr>
<tr>
<td>100</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Source: See Table 1.1.
When $\theta = \mu$, $L = L_1$ the logarithmic variance, and if $\theta = \lambda$ where $\log(\lambda) = (1/n) \sum \log(y_i)$, i.e., $\lambda$ is the geometric mean, then $L = L_2$ the variance of the logs. Both are mean-independent.

One of the most commonly calculated inequality statistics is the Gini coefficient $G$, which is closely related to Figure 1.5. It is equal to the ratio of the area enclosed by the Lorenz Curve and the diagonal line of perfect equality to the total area below the diagonal, or since the latter equals 0.5 (by construction), $G$ equals twice the latter area. It ranges from a maximum of 1.0 (perfect inequality) to zero (perfect equality). Statisticians know $G$ as one half of the relative mean difference—the average of the absolute values of the differences between all pairs of incomes, relative to the mean. Alternatively, think of it as the expected
difference (in a relative sense) between two incomes drawn at random from the income distribution. The formula for $G$ is given by

\begin{equation}
G = \frac{1}{2\mu n} \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j|
\end{equation}

where $|.|$ represents "the absolute value of," but it can also be written as, and more easily estimated from,

\begin{equation}
G = 1 + \frac{1}{n} - \frac{2}{n^2 \mu} \sum_{i=1}^{n} (n - i + 1)y_i,
\end{equation}

remembering individuals are ranked by income so that $y_1$ is the lowest.

An index originally motivated from rather different considerations is Theil's "entropy" index, $T$, where

\begin{equation}
T = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i}{\mu} \right) \log \left( \frac{y_i}{\mu} \right).
\end{equation}

The motivation for this requires a brief excursion into information theory which is concerned with valuing knowledge that an outcome, one of the many possible, has occurred. (This detour is also relevant to section 5.) Suppose there are $n$ mutually exclusive events which may each occur with probability $p_i$, $i = 1, \ldots , n$, with $0 \leq p_i \leq 1$ and $\sum p_i = 1$. If a particular event is very rare, then the information value of a message saying it has occurred is valuable, but the information value of very common events is relatively low. Hence if $h(p_i)$ is the function summarizing value, it will be decreasing in $p_i$. It is also sensible in this context to require that the information content of knowing that two statistically independent events $i$ and $j$ have taken place should be the sum of the values of the separate messages. But since the probability of $i$ and $j$ both occurring is $p_i \cdot p_j$, then it must be that

\begin{equation}
h(p_i \cdot p_j) = h(p_i) + h(p_j)
\end{equation}

and the decreasing function having this property is $h(p_i) = -\log(p_i)$. The expected information content ("entropy" or "disorder") of the whole system is given by the sum of the $h(.)$ weighted by the respective probabilities:

\begin{equation}
\sum_{i=1}^{n} p_i h(p_i) = -\sum_{i=1}^{n} p_i \log(p_i).
\end{equation}

This expression is in effect a measure of the degree to which the probabilities of the various events are equal. The maximum possible value $\Sigma (1/n)h(1/n)$ occurs
when the events are equally likely, and the more disorder—differing probabilities—there is, the more it falls below this. The index subtracting actual from maximum entropy

\[
\sum_{i=1}^{n} \left( \frac{1}{n} \right) h\left( \frac{1}{n} \right) - \sum_{i=1}^{n} p_i \ h\left( p_i \right) = \sum_{i=1}^{n} p_i \left[ \log(p_i) - \log(1/n) \right]
\]

will record higher values the more disorder there is.

The analogy between disorder and income inequality provides the rationale for using such an index in the current context, and if income shares \( s_t = y_t / (n\mu) \), \( t = 1, \ldots, n \) are substituted for the probabilities, \( T \) is the result.

**Properties of Inequality Measures**

Although \( C, L_1, L_2, G, \) and \( T \) each take account of all (changes in) income differences in a distribution, the various formulae indicate that the same information is being weighted differently in the aggregation process. To explore this further, consider some distribution \( A \) from which a person (labelled \( i \)) is arbitrarily chosen. Now form a new distribution \( B \) by transferring a small amount of income to a poorer person \( j \), though keeping \( i \) richer overall. Faced with this situation virtually everyone would agree that the inequality falls in going from \( B \) to \( A \) (though they may disagree about how much), and an inequality measure \( I \) satisfying this property is said to satisfy the Principle of Transfers. For infinitesimal progressive transfers of the type described, this reduces to the condition \( dl < 0 \), where

\[
dl = \left( \frac{\partial I}{\partial y_j} \right) dy_j + \left( \frac{\partial I}{\partial y_i} \right) dy_i = dy \left[ \left( \frac{\partial I}{\partial y_j} \right) - \left( \frac{\partial I}{\partial y_i} \right) \right],
\]

and the change in inequality, \( dl \), is the total differential of \( I \), and note that by construction the transfer \( dy_i = -dy_j \).

Views about the precise size of the inequality reduction from the transfer \( dy \) are likely to depend on the income level of the recipient. Taking two pairs of individuals the same income distance apart, where one pair is relatively rich and the other relatively poor, many would argue that a given transfer from richer to poorer should reduce inequality more for the second pair than the first. (Or in other words, a progressive transfer between the poor pair combined with a regressive transfer of the same size between the rich pair would reduce overall inequality.) Inequality measures satisfying this property are known as transfer sensitive. For transfer sensitivity to hold the fall in inequality from a progressive transfer must be greater (\( dl \) more negative) the lower the income of the recipient is.

Do the measures cited satisfy the principle of transfers? From equations (2)—
one may calculate that the changes in inequality arising from an infinitesimal mean-preserving progressive transfer are given by

\[
\begin{align*}
(11a) \quad dC &= \frac{dy}{n\mu^2 C} [y_j - y_i] < 0 \\
(11b) \quad d(C^2 / 2) &= \frac{dy}{n\mu^2} [y_j - y_i] < 0 \\
(11c) \quad dL &= \frac{2dy}{n^2 \mu} \{ \log(y_j / \theta) - \log(y_i / \theta) \} \leq 0 \\
(11d) \quad dG &= \left[ \frac{2dy}{n^2 \mu} \right] [j - i] < 0 \\
(11e) \quad dT &= \left[ \frac{dy}{\mu} \right] \{ \log(y_j) - \log(y_i) \} < 0.
\end{align*}
\]

These equations show that the Principle of Transfers is satisfied for every possible income pair by all measures, except the logarithmic variance and the variance of the logs, with the effects in different parts of the distribution summarized by the terms in curly brackets.

The coefficient of variation \(C\), and the ordinally equivalent \(C^2/2\) (relevant to section 5), would record exactly the same change if $100 were transferred from someone with $500 to another with $400 (20 percent lower in relative terms), or from someone with $50,000 to someone with $49,900 (0.2 percent lower). The measure is therefore much more strongly affected by relative income differences amongst those with high incomes, than amongst low incomes. It is also not transfer sensitive.

In contrast the Theil index depends on relative rather than absolute income differences and is transfer sensitive. A $100 transfer from someone with $50,000 to someone with $40,000 would change \(T\) by as one from someone with $500 to someone with $400.

The Gini coefficient is rather different, for here the response to transfers depends on the rank orderings of the two persons. The change will be the larger the closer the pair is to the more crowded middle of the distribution (more precisely, the mode) rather than the sparser upper and lower tails. Although the Principle of Transfers is satisfied, the measure is not transfer sensitive because of the dependence on ranks rather than income.

Figure 1.4 showed how a log transformation radically compressed absolute income differences at the top of the distribution, and so one might expect the log-income-based measures \((L_1 \text{ and } L_2)\) to be relatively sensitive to transfers amongst those with low incomes, like Theil’s \(T\). This is indeed true but the problem is that the log income measures satisfy the Principle of Transfers only as long as the poorer person has an income less than \(e\) times the mean \(\theta\) (where \(e\) is the exponential constant 2.718 . . . ). If it is higher than this then \(dL > 0\), which may be checked by examining the properties of the function \((1/z)\log(z)\). The intuition is that at very high incomes the transfer reduces absolute income differences but the compression effect from taking logs overdoes things, to give a perverse effect overall. It is this failure of the \(L\) measures to satisfy the Principle of Transfers that has led to frequent criticism of it as an inequality measure.
The conclusion to be drawn from this analysis is that all inequality measures, even ones related to apparently objective diagrams, inevitably involve value judgments of various kinds, and that some of these implicit assumptions are not necessarily desirable. Subsequent theoretical work has therefore largely followed the strategy of incorporating at the very outset properties seen as desirable.

The first axiom usually used is Symmetry, sometimes known as anonymity, which means that the inequality measure is based only upon the information provided by the incomes in a distribution, and not for example, by who the people are that have particular incomes. This may appear a strong assumption, but it is hardly objectionable as long as appropriate adjustments, such as for differing family sizes, have been made beforehand. This emphasizes the importance of getting the "essential preliminaries" right. All the indices considered so far satisfy Symmetry.

A second axiom, Mean-Independence has already been discussed. The desirability of having inequality measures invariant to proportionate changes in all incomes has been assumed by most writers in the literature and, as will be seen below, plays an important role in characterizing further measures.

A third property is Population Homogeneity which requires that measures be invariant to replications of the distribution; a population formed by merging \( q \) identical populations would be \( q \) times as large but have the same degree of inequality. (The desirability of the axiom is often taken for granted, but it is not absolutely persuasive: is a distribution where one person has \$1\ million and another \$100\ as unequal as one where five have \$1\ million each and five, \$100 each?)

The fourth and most fundamental axiom used is the Principle of Transfers, defined above, which encapsulates the link between a reduction in inequality and (mean-preserving) progressive transfers. The fifth property, Transfer Sensitivity, is sometimes used in tandem with this.

Suppose now that a whole series of transfers from richer people to poorer people are carried out. At each step pair-wise comparisons of distributions using the Principle of Transfers imply that each new distribution is more equal than any older one. It means that if a distribution can be reached from another by a sequence of mean-preserving transfers then it is unambiguously more equal, and this provides a method for ranking distributions. However verifying that such sequences exist is tedious and impractical in reality, and so analysts have searched for conditions that are much more easily checked.

4. Measures for Inequality Rankings

To motivate the results following, consider two distributions of income \( y = (5, 10, 20, 25, 40) \) and \( x = (5, 10, 21, 24, 40) \), where \( y \) has been derived from \( x \) by a single mean-preserving transfer taking \$1\ from the person with \$25\ and giving it to the one with \$20\. Drawing the Lorenz Curves for the two distributions will
reveal that they exactly coincide except between cumulative population shares 40 percent and 80 percent where the curve for \( x \) lies above that for \( y \). Given the Principle of Transfers it follows that \( I(x) < I(y) \), a result consistent with the area-based interpretation of the Gini coefficient above. One can check that also transferring $1 from the person with $10 to the one with $5 has a similar consequence. This suggests that where \( x \) and \( y \) are now any two distributions, the statement that "\( x \) is obtained from \( y \) by a series of mean-preserving progressive transfers," (and hence "more equal" than \( y \)) directly corresponds to "\( x \) has a Lorenz Curve never lying below that for \( y \) and somewhere lying above" (i.e., \( x \) Lorenz-dominates \( y \)). And this has been formally proven:

(T1) For any two arbitrary distributions \( x, y \), with the same number of members and \( \mu_x = \mu_y \), whose members are ranked by income to give ordered distributions \( x' \) and \( y' \) respectively, then \( x' \) is obtained from \( y' \) by a finite sequence of progressive transfers if and only if \( x \) Lorenz-dominates \( y \).

The result is particularly useful because the condition on Lorenz Curves is so easily checked. The problem is, of course, that inequality comparisons will be indecisive wherever Lorenz Curves cross (see the dotted line in Figure 1.5), and the result cannot be applied where the distributions have different means. Before considering some of the strategies adopted as a consequence of this, it will be fruitful to redevelop the results within a different framework.

**Social Welfare, Inequality Rankings, and Lorenz Curves**

Those familiar with welfare economics will know that it is standard practice to evaluate the desirability of policy outcomes in terms of their effects on social welfare. This in turn is measured by using some form of social welfare function (SWF) to aggregate outcomes across all persons in the population. It has been seen in the preceding sections that different inequality measures incorporate different systems for weighting individual incomes, and so in this sense are just like some form of SWF. This suggests that an alternative approach could be to start with a SWF, specified to incorporate certain social values, and then to derive an inequality measure from this, knowing that it must reflect these same properties.

A typical specification of the SWF is

\[
W = \sum_{j=1}^{J} p_j \, U\left(y_j\right),
\]

where \( p_j \) is the proportion of the population with income \( y_j \), and \( U(.) \), the social income valuation function is nondecreasing in each \( y_j \) and strictly concave. Where everyone has a different income this reduces to
MEASUREMENT OF INCOME INEQUALITY

\[(13)\]
\[W = \frac{1}{n} \sum_{i=1}^{n} U(y_i),\]

which, for expositional reasons, is the form used below. It follows that the change in social welfare arising from moving from one distribution to another is given by

\[(14)\]
\[dW = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\partial U}{\partial y_i} \right) dy_i,\]

the sum of the changes in each individual's income \(dy_i\) weighted by a factor \(\frac{\partial U}{\partial y_i}\), the marginal social evaluation.

The assumptions incorporated are, first, that an increase in someone's income must improve social welfare or at worst, leave it unchanged; it cannot fall. Second, everyone's incomes are evaluated using the same \(U(.)\) function, so that \(W\) is symmetric (for the same reasons as discussed previously, and with the same caveat). Third, \(W\) is an additively separable function of the individual incomes. This means that changes in social welfare arising from changing one individual's income are entirely unaffected by those of others. Social recognition of, say, envy \(\frac{\partial U(y_j)}{\partial y_i} < 0\) is thus ruled out.

The fourth assumption, strict concavity, is the key axiom encapsulating the way in which individuals' incomes, and differences between them, are evaluated by society. It says that the welfare weights used in the assessment process are non-negative for each individual \(\frac{\partial U}{\partial y_i} \geq 0, \text{ all } i\), but the higher a person's income is, the lower the weight received \(\frac{\partial^2 U}{\partial y_i^2} < 0, \text{ all } i\), and implies a social preference for equality.

This is shown by Figure 1.6, which illustrates equation (13) for the case where \(n = 2\). Higher levels of \(W\) correspond to higher contour lines (social indifference curves), and the symmetry assumption ensures these are symmetric about the 45° ray from the origin. Strict concavity ensures the curves are bowed towards the origin (they are straight lines like AD when only concavity is assumed). The point B represents the distribution \((y_i, y_j)\). If some income is transferred from the richer person \(i\) to the poorer \(j\), the new distribution must lie at a point somewhere along the line BC (in the limit at point C, where they are equal), and corresponds to a higher level of social welfare.

Assumptions about the degree of concavity of the SWF, or equivalently about the individual welfare weights, see (14), thus correspond to assumptions about the Principle of Transfers. The more averse to inequality society is, the more bowed the contours will be, with the limiting case being where they are the dotted right-angle shape. In this situation progressive transfers reduce inequality only if the income of the very worst-off person is improved.
The assumption corresponding to transfer sensitivity is that the third partial derivative of $U(.)$ be positive. To see this, suppose there is a transfer from $i$ to $j$ of amount $dy$, and this changes social welfare by (from (14))

\[(15) \quad dW = \left( \frac{dy}{n} \right) \left[ \left( \frac{\partial U}{\partial y_j} \right) - \left( \frac{\partial U}{\partial y_i} \right) \right] > 0.\]

Transfer sensitivity requires the term in square brackets to be larger when $i$ and $j$ are both relatively poor than when they are both relatively rich. As Figure 1.7 illustrates, a sufficient condition for this is that the slope of the welfare weighting function ($\frac{\partial^2 U}{\partial y^2}$) becomes less negative as $y$ increases, i.e., $\frac{\partial^3 U}{\partial y^3} > 0$.

Note, finally, that $W$ is invariant to replications of the population.

An important result regarding SWFs can now be stated:

(T2) For distributions $x$, $y$ with associated levels of social welfare $W_x$ and $W_y$ respectively, where $W$ is any additively separable strictly concave non-decreasing social welfare function, and $\mu_x = \mu_y$, then $W_x > W_y$ if and only if $x$ Lorenz-dominates $y$.

This summarizes the intimate relationship between inequality measurement
and social judgments, and can be linked up with T1. Different specific forms might be chosen for $U(\cdot)$, but as long as they are nondecreasing and strictly concave, they will each rank distributions identically. As an illustration, suppose $U(y_i) = (y_i - \mu)^2/2\mu^2$ or $(y_i/\mu)\log(y_i/\mu)$, and substitute these into equation (13). The theorem implies that $C^2/2$ and $T$ are ordinally equivalent measures.

**Extensions to the Results**

The result summarized in T2 is restricted to comparisons of distributions with equal means, and even then there may be situations where Lorenz Curves intersect so that unambiguous conclusions cannot be drawn.

When means differ, one approach is to persist in ranking the distributions by the Lorenz criterion which is equivalent to assuming that the inequality measure implicit in the analysis is mean-independent. In fact it can be shown that:

(T3) The class of inequality measures satisfying Symmetry, Mean-Independence, Population Homogeneity, and the Principle of Transfers simultaneously is exactly equivalent to the class satisfying the property that a higher (the same) Lorenz Curve implies less (the same) inequality.
This is the key theorem, for it gives the conditions under which all standard relative inequality measures will agree in their inequality rankings.

Recent work has attempted to widen the scope for deriving unambiguous rankings by different routes. One strategy has been to strengthen the Principle of Transfers and require Transfer Sensitivity as well. Stronger assumptions narrow the class of inequality measures (and SWFs) considered but the trade-off is that dominance requirements are reduced. Results giving the modified conditions under which one may say two distributions can be ranked unambiguously have been derived in the literature, but unfortunately they are not at all easy to interpret, and there is no straightforward check available corresponding to Lorenz Curve comparisons. However the analysis has proved particularly useful in providing a condition for situations where the earlier weaker assumptions lead to inconclusive results:

(T4) If the Lorenz Curve for \( x \) intersects that for \( y \) once from above, then

(a) \( \mu_x = \mu_y \), \( I(y) > I(x) \) for all transfer-sensitive inequality measures \( I(.) \), i.e., \( W(x) > W(y) \), if and only if \( V(y) \geq V(x) \), or

(b) \( \mu_x \neq \mu_y \), \( I(y) > I(x) \) for all transfer sensitive inequality measures that also satisfy Mean-Independence and Population Homogeneity if and only if \( C(y) > C(x) \).

Since multiple Lorenz Curve crossings are rare this is very helpful. Where there is a single crossing and equal means, just a simple inspection of variances may resolve the previous impasse. When the means differ, checks can still be done as long as two additional but not implausible assumptions are invoked. Then the comparison is of the coefficients of variation.

An alternative approach is to rank distributions in terms of social welfare rather than just inequality per se, by incorporating information about average living standards into the analysis as well.

Take the distributions \( x \) and \( y \) again and suppose now that \( \mu_x > \mu_y \). Raising all incomes in \( y \) proportionately up to the point where the new mean equalled \( \mu_x \) would raise social welfare since \( W \) is nondecreasing in each income. But, by definition, the Lorenz Curve for \( y \) would be unchanged, and so by T2 the scaled-up \( y \) distribution would still not be socially preferred to \( x \). Thus the \( \mu_x = \mu_y \) phrase in T2 can be replaced by \( \mu_x \geq \mu_y \)—clearly, social welfare increases if there is less inequality and average income is higher.

If average income is lower (\( \mu_x < \mu_y \)), unambiguous rankings are more difficult to derive because society’s equality preference (built into \( W \) by the concavity assumption) conflicts with efficiency preference (the desire for higher incomes, ceteris paribus, built in by making \( W \) nondecreasing). On the other hand it is also intuitively plausible that there could be circumstances where if \( \mu_y \) were sufficiently high then this could offset having a lower Lorenz Curve over parts of (or all) the population. Making some of the poor poorer might be tolerated if average living standards amongst all poor people were raised by enough. Deriving ranking criteria taking account of this idea is all the more important given that in
practice differences between Lorenz Curves are relatively small compared to
differences in means.

Before stating the required result it will be necessary to revise the definition
of a Lorenz Curve: instead of plotting cumulative income shares against cumula­
tive population shares, now plot cumulative mean incomes,

\[
\frac{1}{n} \sum_{i=1}^{k} y_i, \text{ for each value of } k = 1, \ldots, n.
\]

This gives the \textit{Generalized Lorenz Curve}, which is the ordinary Lorenz Curve
with each point scaled vertically by mean income. The theorem may now be
stated.

(T5) \textit{For all additively separable nondecreasing strictly concave SWFs,} \(x\)
\textit{yields a higher (the same) level of social welfare than} \(y\) \textit{if and only if the
Generalized Lorenz Curve for} \(x\) \textit{lies above (on) that for} \(y\).

This represents a notable strengthening in the ability to make unambiguous
comparisons of distributions. However it needs to be remembered that the under­
lying yardstick is overall social welfare, not inequality per se. Whether this is
appropriate will depend on the particular goal of the analyst.

\textbf{Analogies with Decision Making under Uncertainty}

Many of the results summarized above have been inspired by the literature on
decision making under uncertainty, and it is useful to draw out this link in more
detail.

Ranking income distributions in terms of social welfare is a problem with
exactly the same structure as ranking probability distributions in terms of their
expected utility. It is as if an individual is to be given an income drawn at
random from one of two distributions, where the probability \(p_i\) of getting any
particular value is known (but not, in advance, which specific income would be
received ex post), and the individual has to choose which distribution the draw
will be from. If the means of the distributions are the same, the decision will
depend both on the relative risks involved (related to the differences in probabili­
ties) and on attitudes to risk (which are related to the concavity of \(U(.)\) if this is
now interpreted as the individual's utility function). A choice based on maximiz­
ing expected utility is then exactly analogous to maximizing \(W\) as given in (12).
Aversion to risk is like aversion to inequality.

But in the uncertainty literature there is a well-known result which says that
for two distributions with the same mean, and all increasing concave \(U(.)\), then \(x\)
will be unambiguously preferred to \(y\) if and only if \(x\) second order stochastically
dominates \(y\). Second order stochastic dominance can be explained with reference
to Figure 1.3(b). Let the curve shown be that for \(y\) and suppose another is added
for \( x \) with the same general shape. If this new curve is everywhere on or below that for \( y \) then \( x \) is said to first order stochastically dominate \( y \). Now suppose the curves intersect (possibly several times), and evaluate the areas between the curves, cumulatively for each income share along the horizontal axis, using the convention that where the curve for \( x \) lies below that for \( y \) the area is negative. Second order stochastic dominance is the condition requiring that the cumulative areas are never strictly positive, and are strictly negative at least one point. This condition is difficult to handle (even for decision theorists!), but fortunately it can be shown that the statement "\( x \) second order stochastically dominates \( y \)" is exactly equivalent to "\( x \) Lorenz-dominates \( y \)". \( T1 \) and \( T2 \) are thus further extended. Intuitively, the result arises from the close link between Figures 1.3(b) and 1.5. The area under the graph and to the left of a given point on the horizontal axis of Figure 1.3(b) corresponds to a point on the vertical axis of Figure 1.5. And both figures record population shares on their other axis.

Note, finally, that Transfer Sensitivity is analogous to decreasing absolute risk aversion. Making this assumption, \( x \) is preferred to \( y \) if and only if \( x \) third order stochastically dominates \( y \). Checking this condition requires a further cumulation of areas and is not straightforward. The role for the variance summarized in \( T4 \) is therefore very useful.

5. The Atkinson and Generalized Entropy Families of Inequality Indices

The easily-implementable checks discussed above may not resolve ambiguities when applied and, anyway, one may also want to say whether a given change in inequality is larger than another, or to decompose total overall inequality into constituent components. In these situations numerical measures are required.

The Atkinson family of indices is derived by making additional assumptions about the functional form of the SWF welfare weights and hence relationships between transfers and changes in inequality. The Generalized Entropy (GE) family in contrast is developed by considering these connections directly, and can be interpreted as making assumptions about how distances between individuals' income shares are measured.

**The Atkinson Family of Inequality Indices**

Consider Figure 1.6 again. Remembering that point B represents a position of inequality and point C one of perfect equality consistent with redistributing the same aggregate income total, a natural way to measure inequality is to use the relative difference in social welfares possible at the two points, i.e.,

\[
I = (W_1 - W_0)/W_1 = 1 - (W_0/W_1).
\]
The standard assumption made about $W(.)$ in addition to those discussed above is that the social income evaluation functions $U(.)$, and hence $W(.)$, have constant elasticity. Multiplying every individual's income by a certain proportion would change total social welfare by the same fraction, and this would leave the degree of inequality measured using equation (16) unchanged. This assumption therefore ensures that the measure is mean-independent. (In Figure 1.6 the slopes of the social indifference curves at each point along a ray from the origin will always be the same.)

For this to be true $U(.)$ must have the form

\[ U(y) = a + by^{1-\varepsilon} / (1 - \varepsilon) , \quad \varepsilon \neq 1, \quad \varepsilon \geq 0 \]

\[ = \log(y), \quad \varepsilon = 1. \]

where, by analogy with the literature on uncertainty, $\varepsilon$ may be interpreted as the constant degree of relative inequality (rather than risk) aversion. The restriction $\varepsilon \geq 0$ ensures the $W$ contours are concave to the origin with the higher the value of $\varepsilon$, the more bowed they are ($\varepsilon = 0$ is the straight line case in Figure 1.6). The welfare weights implied are

\[ \frac{\partial U}{\partial y} = by^{\varepsilon}, \quad \varepsilon \neq 1, \quad \varepsilon \geq 0 \]

\[ = \frac{1}{y}, \quad \varepsilon = 1 \]

which have graphs with the same shape as Figure 1.7—higher values of $\varepsilon$ corresponding to steeper-sloped curves—which means that measures derived will be transfer sensitive.

One could now derive inequality measures by simply substituting equation (17) into (16), but a problem with this is that the number calculated would depend not just on $\varepsilon$, but also on $a$ and $b$, as arbitrary constants. (Variations in the constants would provide a series of ordinally equivalent measures.) However this may be fixed by introducing the concept of the equally distributed equivalent level of income ($y_e$), which is the amount of income which if equally distributed amongst all persons would give the same level of social welfare as the original distribution, i.e., $y_e$ such that

\[ W = \frac{1}{n} \sum_{i=1}^{n} U(y_i) = \frac{1}{n} \sum_{i=1}^{n} U(y_e) = U(y_e) \]

and so, from (17)

\[ y_e = \left[ \frac{1}{n} \sum_{i=1}^{n} y_i^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \]
which does not depend on \( a \) or \( b \). The measure of inequality is redefined as

\[
I = 1 - \frac{y}{\mu},
\]

which corresponds to the (relative) distance \( EC/OC \) in Figure 1.6, and ranges between zero (perfect equality) and one (perfect inequality). In risk analysis, \( \mu - y \) is the maximum value a risk-averse decision maker would be willing to pay to swap a risky choice for a riskless one with equal expected value.

Substituting equation (20) into equation (21) gives the Atkinson family of indices:

\[
A_\varepsilon = 1 - \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i}{\mu} \right)^{(1-\varepsilon)} \right]^{1/(1-\varepsilon)}, \quad \varepsilon \neq 1, \varepsilon \geq 0
\]

\[
= 1 - \exp \left\{ \frac{1}{n} \sum_{i=1}^{n} \log_e \left( \frac{y_i}{\mu} \right) \right\}, \quad \varepsilon = 1.
\]

where \( \exp[.] = e^{(.)} \).

Equation (21) shows that estimated values have a clear intuitive interpretation in terms of the gains from redistribution. A value of say 0.3 (= 1 - 0.7) indicates that if incomes were equal, then only 70 percent of current total income would be required to achieve exactly the same level of social welfare. Policies having both efficiency effects (changing total income) and redistributive ones can be explicitly compared. For example one could say that the equalizing effect of some pro-poor social security reforms were equivalent to a certain percentage increase in equally distributed income, a measure of social welfare. It is sometimes argued, rightly or wrongly, that efficiency enhancing policies have adverse distributional effects. Equation (21) implies \( y = \mu(1-f) \), an expression which brings out the policy tradeoff between efficiency (\( \mu \)) and equality (\( 1-f \)) explicitly.

Of course answers will depend on the particular SWF used or, equivalently, the degree of risk aversion, \( \varepsilon \). How might this be chosen? A tax-transfer policy that paid \$1 to some poor person \( P \) via a tax of \$1 on richer person \( R \) would certainly be approved by all inequality-averse people (those with \( \varepsilon \geq 0 \)). But many would endorse giving \$1 to \( P \) even if it meant that \( R \) was taxed by more than this and the extra was "lost" (for example in administrative costs). Suppose \( R \) has four times the income of \( P \), then from equation (15) the maximum amount society would be willing to tax \( R \) to ensure \( P \) got \$1 is \$4^\varepsilon \). Hence, when \( \varepsilon = 0 \) the tax on \( R \) would be \$4^0 = \$1.00

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>tax on ( R ) would be</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>$4^{0.25} = $1.41</td>
</tr>
<tr>
<td>0.5</td>
<td>$4^{0.5} = $2.00</td>
</tr>
</tbody>
</table>
1.0 the tax on $R$ would be $4^1 = $4.00
2.0 the tax on $R$ would be $4^2 = $16.00
4.0 the tax on $R$ would be $4^3 = $256.00.

With an infinitely large $\varepsilon$ there is no limit to the amount; improvement of the position of the very poorest person is given absolute priority in this case.

For an alternative interpretation, suppose now that the top 5 percent of the income distribution all have incomes equal to $y_R$, and each of the bottom 20 percent, $y_p$, so that a tax of $1000$ on each of the former could, if costlessly transferred, provide $250$ per capita for the latter group. But in practice the "bucket" carrying the money from the rich to the poor leaks during the transfer process. The amount of leakage one would tolerate and yet still be in favor of the redistribution also gives clues about the choice of $\varepsilon$. With the distribution described, those with $\varepsilon = 1$ would allow up to 75 percent spillage; with $\varepsilon = 0.5$, 50 percent; with $\varepsilon = 0.25$, 33 percent; and with $\varepsilon = 0$, none at all.

Larger values of $\varepsilon$ give greater emphasis to redistributions reducing income differences at the bottom of the distribution relative to those at the top (see Figure 1.7), with measures becoming very bottom-sensitive for values greater than one.

**The Generalized Entropy Family of Inequality Indices**

Rather than strengthening the discriminatory power of the theorems of section 4 indirectly, by specifying SWFs and imposing plausible restrictions on these, the main alternative approach is an axiomatic one: it specifies at the outset a set of desirable properties for the measure itself to have directly, and then uses these to characterize the index. The formulae derived are not always easy to rationalize intuitively, but here follows one quite plausible interpretation.

Focus attention on the relationship between the Principle of Transfers and changes in inequality. Equation (11) summarized this link for a range of measures and it was noted that a minimal requirement for the Principle of Transfers to be satisfied was $dl < 0$, and the size of the redistributional effect varied with the $\{\cdot\}$ term.

Suppose now that the Principle of Transfers is modified to require not only $dl < 0$ but also that it depends only on the income shares of $i$ and $j$. The transfer effect cannot then depend on their ranks (as with $G$), and must be independent of all other persons' income shares. The two measures $T$ and $C^2/2$ certainly satisfy this Strong Principle of Transfers, since equations (11b, 11e) can be rewritten as

\begin{align*}
(23a) \quad d(C^2/2) &= (dy/\mu)(s_j - s_i), \text{ and} \\
(23b) \quad dT &= (dy/\mu)(\log(s_j) - \log(s_i))
\end{align*}

where $s_t = y_t/(n\mu)$ is $t$'s income share, $t = i, j,$ and moreover there is a very close
similarity in the way the transfer effect is quantified. The key element (in curly brackets) is simply the absolute difference between \( i \)'s and \( j \)'s income shares for \( C^2/2 \), while for \( T \) it is based on relative income differences. Each measure incorporates a different concept of "distance between income shares," which suggests that a family of inequality measures could be built up by developing the distance idea. What the "generalized entropy" approach does is generalize the notion of distance used in equation (9), with the rationale that there seems little reason for imposing the restriction implied by equation (7) in the income distribution context.

Taking the \( \{.\} \) term to represent distance between income shares, then those used in equations (23a, 23b) are both special cases of the more general form

\[
\{ h (s_i) - h (s_j) \} \text{ where } \]

\[
h (s_i) = (-1 / \beta) s_i^{\beta}, \beta \neq 0
\]

\[
= -\log(s_i), \beta = 0, \text{ and } i = i, j.
\]

Note that \( \{.\} < 0 \) whatever the \( \beta \), so the Principle of Transfers is always satisfied.

Views about sensitivity to income transfers are thus incorporated directly, with smaller and more negative values of \( \beta \) giving increasing emphasis to income differences at the bottom of the distribution (by lengthening the distance). The parameter \( \beta \) appears to play an analogous role to \( \varepsilon \) and the distance function to the welfare weights function. The relationship is brought out by Figure 1.8, which graphs \( h(s) \) against \( s \), showing figures bowed towards the origin—as in Figure 1.7—for all \( \beta < 1 \). It may be checked that the cases \( \beta > 1 \) are those for which the measure is not transfer sensitive \( (\partial^2 h / \partial s^2 \leq 0) \).

If the specification of distance in equation (24) is used, then the inequality measure that naturally generalizes equation (9) is (for \( \beta \neq 0 \) or \(-1\))

\[
I_\beta = \left[ 1 / (1 + \beta) \right] \sum_{i=1}^{n} s_i \left[ h (1 / n) - h (s_i) \right].
\]

\[
= \left[ 1 / (\beta^2 + \beta) \right] \left[ (\sum_{i=1}^{n} s_i^{\beta + 1}) - n^{-\beta} \right].
\]

This does not satisfy "Population Homogeneity" (except when \( \beta = 0 \)); note the \( n^{-\beta} \) term. However a measure that does, and which has exactly the same \( \{.\} \) terms in the transfer equation as \( I_\beta \), is \( n^\beta \cdot I_\beta \), and it is the latter which is referred to as the Generalized Entropy family. The formula, for \( \beta \neq 0 \) or \(-1\), is given by

\[
E_\beta = \left[ 1 / (\beta^2 + \beta) \right] \left[ (1 / n) \sum_{i=1}^{n} (ns_i^{\beta + 1}) - 1 \right]
\]
Figure 1.8. Alternative Distance Functions, $h(s)$

The diagram shows various distance functions $h(s)$ for different values of $\beta$. The functions are plotted against the income share $s$, with $\beta$ values ranging from $-2$ to $0.5$. Each curve represents a specific value of $\beta$, indicating how the distance function changes with varying income shares.
which may be rewritten, setting $\alpha = \beta + 1$, as

$$E_\alpha = \left[ \frac{1}{\alpha^2} \right] \left[ \frac{1}{n} \sum_{i=1}^{n} (y_i / \mu)^\alpha \right] - 1$$

(27)

For the cases where $\alpha = 0$ ($\beta = -1$) and $1$ ($\beta = 0$), the formulae are

$$E_0 = \frac{1}{n} \sum_{i=1}^{n} \log(y_i / \mu), \text{ and } E_1 = \frac{1}{n} \sum_{i=1}^{n} \log(y_i / \mu).$$

(28)

Note that $T = E_1$ and $C^2/2 = E_2$.

Furthermore, suppose $\alpha = 1 - \varepsilon$, in which case equations (22) and (27) show that

$$A_\varepsilon = 1 - [(\alpha^2 - \varepsilon)E_\alpha + 1]^{1/\alpha} , \alpha < 1, \alpha \neq 0$$

$$= 1 - \exp(-E_\alpha), \alpha = 0$$

(29)

and so for every member of the Atkinson family there is a corresponding, ordi-
nally equivalent, member of the GE family.

What then are the main differences between the two families? First and most
obviously, there are values of $\alpha$ corresponding to $\varepsilon < 0$ for which $A_\varepsilon$ is not
defined, apparently giving the GE family a greater range of measures. On the
other hand, to some, the fact that $E_\alpha$ is not transfer sensitive for $\alpha > 2$
may mean this extension is of little significance.

Second, while the minimum value of each index is zero (when there is com-
plete equality), $E_\alpha$ does not have a maximum of one; the maximum depends on
population size (for example the maximum for $E_1$ is $\log(n)$). This is not necessarily
a bad thing though. It also needs to be remembered that even if an index does
range between zero and one (or is normalized to do so), there is not a linear
inequality scale in between. A value of 0.5 for example could be derived from
very different distributions depending on the measure used.

Third, there is a major advantage of the GE family not shared by the Atkinson one
(or the Gini coefficient for that matter). This is that it is very straightforwardly
additively decomposable by population subgroup. This can be explained as follows.

Suppose everyone in the population is sorted into $M$ mutually exclusive
groups according to some characteristic. A decomposable index is one where the
overall degree of inequality can be calculated from (only) the number of mem-
bers, mean income, and inequality value for each and every subgroup, and has
the property that an increase in inequality in one subgroup must raise overall
inequality. An index is additively decomposable when it can also be written as a
weighted sum of the group inequality indices (the weights are all positive), plus a
between-group inequality term based on mean incomes and group size. It can be shown that:

(T6) The GE family encompasses all possible indices simultaneously satisfying Symmetry, Mean-Independence, the Strong Principle of Transfers and Additive Decomposability. If the latter property is relaxed to Decomposability, the index must be some positive transformation of the GE family.

Hence, for example, $T$ and $C^2/2$ are additively decomposable and the Atkinson family members only decomposable. The Gini coefficient is neither: with this measure it is possible for inequality within one group to increase and yet overall inequality to decrease!

The disaggregated version of the formula for $E_\alpha$ illustrating additive decomposability is given by:

\[
E_\alpha = \sum_{m=1}^{M} (v^\alpha_m) (w_m^{1-\alpha}) E_{am} + E_{aB}
\]

where the population share of a given group $m$ is $w_m = n_m/n$; mean income within the group, $\mu_m$; and the share of total income held by $m$'s members is $v^\alpha_m = w_m \mu_m^{\alpha}/\mu$. The within-group inequality terms $E_{am}$ are calculated as if each group were a separate population, while the between-group inequality term is derived by assuming every person within a given group receives the group's mean income:

\[
E_{aB} = \frac{1}{(\alpha^2 - \alpha)} \left[ \sum m w_m (\mu_m/\mu)^\alpha \right] - 1, \quad \alpha \neq 0, 1;
\]
\[
E_{OB} = \sum m w_m \log (\mu/\mu_m), \quad \text{and} \quad E_{1B} = \sum m v_m \log (\mu_m/\mu).
\]

There is an obvious attraction in also requiring that the aggregating weights $(v^\alpha_m)(w_m^{1-\alpha})$ on the $E_{am}$ add up to one, which means they reduce to the group population shares for $\alpha = 0$, and group income shares for $\alpha = 1$. Amongst other things, this means that if inequality were to increase by the same proportion in every group, with everything else (including group means and income shares) staying the same, total inequality increases in the same proportion.

The additive decomposability property is very important for empirical work on the structure of inequality and changes in it. For example one may compare inequality within and between groups of individuals partitioned according to their age to draw conclusions about the relationship between age and inequality. If age had a very small association with inequality one would expect the inter-group component to be very small and the intra-group ones of a similar size. It is also straightforward to investigate the relative contributions of changes in the age structure and within-group inequality to changes in total inequality over time. Obviously the technique is also applicable to decompositions by other variables such as family composition, geographical location, or principal source of income.
received. It should be noted however that the appropriate method for considering the contribution of different income sources (such as primary and secondary earnings, taxes, pensions) to total inequality—decomposition by factor components—is an entirely different issue and can be shown to be independent of the choice of inequality measure.

6. Concluding Comments

The process of inequality measurement may be broken down into three main steps: (i) preparation of the distribution to be analyzed; (ii) choice of inequality measure; and (iii) calculations and assessment of results. This chapter has focused on the theoretical issues involved with the first two of these, one of the principal goals being to draw attention to the normative character of many of the assumptions involved. For example, important issues raised in section 2 are those of how to standardize incomes to take account of differences in household composition and how to weight each income unit. The underlying theme of subsequent sections is that it is better to use measures that at the outset measure (sets of) social values in a consistent way. This suggests increased utilization of indices from the Atkinson or Generalized Entropy family and relatively less emphasis on commonly used indices such as the Gini coefficient. Moreover there are also easily implementable checks available which often enable conclusions about inequality to be drawn even when there is no agreement about which particular index should be used.

Theoretical issues in the third measurement stage have not been discussed at all, and there is space here only to mention briefly the most important of these. The first arises from the fact that distributational data have often been available to researchers in grouped form, giving only the numbers of people with incomes falling within particular ranges (as in Table 1.1) plus possibly average income within each range. There has therefore been much written on "interpolation" methods, i.e., optimal ways of estimating inequality measures given the limited information. This issue is becoming of less importance as analysts increasingly have access to microdata sources with information available for each income unit. Second, in practice, individuals may sometimes have recorded incomes that are negative or zero if the accounting period is short or if pretransfer incomes are used. None of the measures discussed are designed to cope with negative incomes, and measures requiring log incomes cannot be applied to units with zero income. The standard procedures used are either to omit the troublesome units from the analysis, or to allocate each of them a very small positive income, say $1. Neither procedure is particularly satisfactory and each may give very different answers. (Note that this problem is much more serious when measuring the inequality of wealth, since quite a few households have liabilities in excess of assets or no net assets.) Third, virtually all data sets are based on samples rather than complete populations, and so estimates of inequality measures are subject to
sampling errors. It is possible for apparent differences in equality not to be statistically significant. This is an issue that has only rarely received attention in the past, but it should receive increasing emphasis as theoretical results become more widely known and the checks implied by them more easily implemented.

7. Further Reading

This section provides a very selective annotated guide to the theoretical literature on inequality measurement. More wide-ranging and comprehensive discussions are given in the books by Atkinson (1983a), Cowell (1977), and Sen (1973). (Section 2 follows closely the structure of Atkinson's chapter 3, and sections 3-5, Cowell's chapters 2 and 3.) At a more technical level, see the books by Nygård and Sandström (1981) and Lambert (1989), and the survey article by Foster (1985). Each of these has extensive bibliographies. The distinction between inequality and poverty is discussed by Atkinson (1987), though note that the modes of analysis used to analyze each topic increasingly overlap (as this chapter and the one by Hagesaar illustrate).

Comprehensive definitions of income are much discussed in the taxation literature; two critiques of approaches adopted in the recent U.S. tax reforms are Pechman (1987) and Musgrave (1987). The relationship between income variability, length of time period, and inequality is analyzed by Shorrocks (1978). Methods for estimating lifetime earnings and thence inequality are considered by Creedy (1977) and Irvine (1981). Paglin (1975) discussed lifetime variations but his methods have been much criticized and they have now been superceded by approaches based on decomposable indices; see Cowell (1984). The unit of analysis is discussed by Morgan et al. (1962). Deaton and Muellbauer (1980) provide the best available survey of equivalence scales, but they should be read alongside the papers by Pollak and Wales (1979) and Fisher (1987) which emphasize normative aspects. The different ways of weighting units are considered by Danziger and Taussig (1979) and Cowell (1984).

All elementary statistics texts have sections on summarizing distributions, but Pen's Parade is best described in its originator's own very evocative terms; see Pen (1971). Dalton's (1920) classic article anticipates by half a century the modern approach of analyzing inequality measures in terms of desired properties. Subsequent landmarks include the papers by Kolm (1969, 1976) and Atkinson (1970). Kolm also provides the most well-known defense of translation independence (equal absolute, not proportional, changes in income leave inequality unchanged). Atkinson's article is the most cited in the inequality measurement literature. Drawing upon analogies with the uncertainty literature it demonstrated the links between sequences of progressive transfers, changes in social welfare, and the crossing of Lorenz curves, summarized in T1 and T2. Subsequent work showed that the relationships still hold if a SWF concavity assumption weaker than that cited in this chapter is used (Dasgupta, Sen
and Starrett 1973). The "leaky bucket" interpretation of the inequality aversion parameter is taken from Okun (1975, chapter 4), whose book is a most eloquent discussion of the tradeoff between equality and efficiency and the role of the market vs. the state. T3 is taken from Foster (1985); see also Fields and Fei (1978). The transfer sensitivity axiom and the role of the variance in extending comparisons of previously noncomparable distributions summarized in T4 are discussed by Shorrocks and Foster (1987), while the properties of Generalized Lorenz Curves, and T5, are set out by Shorrocks (1983). The SWFs implicit in a wide range of inequality measures are derived, and graphed for \( n = 3 \), by Blackorby and Donaldson (1978).

Index T and its entropy interpretation are given by Theil (1967), while its distance-between-income-shares generalization is derived in papers by Cowell and Kuga (1981a, 1981b). Results demonstrating the relationship between the Generalized Entropy family and decomposability by population subgroup are proved by Bourguignon (1979), Cowell (1980), and Shorrocks (1980) and more generally by Shorrocks (1984). Shorrocks's (1982) paper is the leading theoretical analysis of procedures for inequality decomposition by factor components.

A classic article on the estimation of the Lorenz Curve and the Gini coefficient from grouped data is Gastwirth (1972), but Cowell and Mehta (1984) provide the current state-of-the-art analysis of interpolation methods for a much wider range of measures. The most easily accessible discussion of standard errors and tests of significance for inequality measures is in chapter 5 of Cowell (1977), but even this is frustratingly brief. Easy ways of calculating the indices discussed in this chapter from large microdata sets using standard computer packages such as SPSSX are given by Jenkins (1988).

References


The Measurement of Income Inequality


Cross-National Comparisons of Inequality and Poverty Position


Global Economic Inequality and Its Trends Since 1950


4 The Distribution of Household Wealth: Methodological Issues, Time Trends, and Cross-Sectional Comparisons


The Definition and Measurement of Poverty


Fuchs, V. (1967). "Redefining Poverty and Redistributing Income." Public Interest (Summer), 88-95.


Short- and Long-Term Poverty in the United States: Measuring the American Underclass


### Social Security and Income Redistribution


The Impact of Government Tax and Expenditure Programs on the Distribution of Income in the United States


Conclusion


