As different laser technologies continue to make it possible to change laser parameters and improve beam quality and performance, a multidisciplinary theoretical knowledge and grasp of cutting-edge technological developments also become increasingly important. The revised and updated Laser Technology, Second Edition reviews the principles and basic physical laws of lasers needed to learn from past developments and solve the many technical problems arising in this challenging field.

The first edition of Laser Technology was classified by the Chinese National Education Committee as a “national-level key textbook.” This edition presents the fundamentals of physical effects in technical devices and implementation methods to create a clear and systematic understanding of the physical processes of different laser technologies.

Logically presenting the various types of laser technology currently available, this updated second edition:

• Explores the transmission of information using optical waves with modulating technology
• Shows how beam energy or power can be greatly enhanced through Q switching, mode-locking, and amplification
• Explains how mode selection and frequency stabilizing technology make it possible to improve light beam directionality or monochromaticity
• Describes nonlinear optical technology that helps obtain new frequencies and light waves
• Covers transmission in the atmosphere and underwater

Technical improvements to enhance laser performance in different applications have given rise to new physical phenomena. These have resulted in a series of new laser branches and fields of applied technologies, such as laser physics, nonlinear optics, laser spectroscopy, laser medicine, and information optoelectronic technology. This book analyzes this growth, stressing basic principles but also including key technical methods and examples where needed to properly combine practical and theoretical coverage of this distinct area.
Preface

Since the 1980s, this textbook has been examined and recommended for publication by four sessions of the Educational Steering Committee and was designated by the National Education Committee in 1997 as the National Level Key Textbook of the 9th Five-Year Plan period. Revised on the basis of the first edition, the second edition was designated as the National Level Planned Textbook of the 10th Five-Year Plan period by the Ministry of Education in 2001.

The Editor-in-Chief of this textbook is Professor Lan Xinju with Huazhong University of Science and Technology and the Deputy Editor-in-Chief is Zhu Changhong.

This course is allotted 70 reference class hours, with the whole book divided into 5 parts (8 chapters). Part 1 (Chapter 1) deals with the laser modulation and deflection technology, and is mainly a discussion on the fundamentals and technologies of electro-optics and acousto-optics; Part 2 (Chapters 2–4), Q modulation (Q-switching), deals with ultrashort pulse and amplifying technology, and is mainly a discussion on the basic theories and implementation methods of increasing the power and energy of laser pulse; Part 3 (Chapters 5–6), mode selection technology, deals with frequency stabilizing technology, with emphasis on the physical principles and implementation methods of single mode (transverse, longitudinal) output and stabilizing the oscillating frequency; Part 4 (Chapter 7) deals with nonlinear optical technology by first explaining the physical concept of nonlinear optics, followed by a discussion on the basic principles and methods of implementation of nonlinear optics with the frequency-doubling (second-harmonic-generation) technology as the focus of attention; and Part 5 (Chapter 8) deals with laser transmission technology, and is mainly a discussion on the theory and technology of optical fiber transmission, with a brief introduction to the transmission technology in the atmosphere and underwater.

Compiled in accordance with the syllabus of a specialized basic course in optoelectronic information in the category of electronic information of engineering for institutions of higher education, this textbook is particularly suitable for readers with a basic knowledge about physical optics and the fundamentals of laser. It can also be used as a textbook by undergraduates in optoelectronic technology, optical information technology, technological physics, optoelectronic instrumentation, and applied physics; the principal reference material for graduate students in physical electronics and related subjects; as well as reference material for faculty and students in relevant specialties or engineers and technicians in optoelectronic technology. As the contents of the chapters in this book are basically independent of each other, there is great flexibility in its reading. Hence convenient selection of the parts for use by specific institutions. In addition, despite the fact that stress is placed on the description of basic principles, a definite number of technical methods and examples have been included in certain chapters or sections to facilitate combination of theory with practice since this is a course of rather distinct practicability. Exercises and questions for further consideration are attached to the end of each chapter.

Chapters 1 and 6 are written by Lan Xinju of Huazhong University of Science and Technology, Chapters 2 and 5 by Chen Peifeng of the same university, Chapter 3 by Yao Jianquan and Ning Jiping of Tianjin University, Chapter 4 by Liu Jingsong of Huazhong University of Science and Technology, Chapter 7 by Zhu Changhong of the same university, and Chapter 8 by An Yuying of Xi’an University of Electronic Science and Technology, with Lan Xinju responsible for the whole manuscript. In the course of compilation of this book, not all the
works and articles by other authors referred to have been listed in our references one by one. The authors take this opportunity to extend to them our sincere thanks. Owing to our limited scholastic level, criticisms on hard-to-avoid drawbacks and even mistakes are cordially invited.

Editors
Introduction

As one of the major inventions of the 20th century, the laser is playing a unique role in all facets of modern science and technology because of its light emitting mechanism different from that of common light sources and its unusually excellent characteristics, such as very good directionality, high luminance, and good mono-chromaticity and coherence. The impact is keenly felt in industrial, military, telecommunication, medicine, and scientific research. Since the appearance of the first laser in 1960, a diversity of different types of laser, have been invented one after another in the subsequent 40-odd years, mainly including solid lasers such as the Nd:YAG laser, the Nd glass laser, and the ruby laser; gas lasers such as the He-Ne laser, the Ar+ laser, and the CO2 laser; semiconductor lasers such as the GaAs laser and the In-GaAsP/InP laser; quasi-molecular lasers such as the XeCl laser and the KrF laser; and free electron lasers. It can be said that each of the great variety of lasers has its own peculiar performance different from others. However, the physical attributes possessed by all lasers are basically fixed and it’s impossible for all the above-mentioned characteristics to be ideal. Often, the laser output from an ordinary laser may not necessarily satisfy the requirements of certain applications. In this case, in order to meet the needs of different actual applications, laser techniques intended to improve and enhance the performance of lasers have steadily been investigated and developed with R&D concerning the interaction between laser and material going on at the same time. With these techniques available, laser application has greatly expanded—so much so that a number of brand new physical phenomena have occurred, forming a series of new laser branches and fields of applied technologies, for example, laser physics, nonlinear optics, laser spectroscopy, laser medicine, and information optoelectronic technology.

It is obvious that an ordinary pulse solid laser, with an output optic pulse width of the order of several hundred μs or even ms and a peak power of the level of dozens of kW, is definitely incapable of meeting the requirements of research on precision ranging with laser, laser radar, high-speed photography, high-resolution spectroscopy, to name but a few. It was against such a background that the laser Q modulating technology and mode locking technology were investigated and developed. As far back as 1961, shortly after the laser made its debut in 1960, the concept of Q modulation was put forward, that is, it was assumed that a method could be adopted to compress all the optical radiation into an extremely narrow pulse for emission. In 1962, Hellwarth and Mcclung made the first Q modulation laser with an output peak power of 600 kW and a pulse width of the order of 10^{-7} s. The development in this area was very rapid in the subsequent few years until there appeared a multitude of Q modulating methods, such as electro-optic Q modulation, acousto-optic Q modulation, and saturable absorption Q modulation, with the output power almost abruptly rising. Great progress was also made in pulse width compression. By the 1980s, it was no longer difficult to generate giant pulses with a pulse width of the ns order and a peak power of the GW order using the Q modulating technology. The emergence of the Q modulating technology is an important breakthrough in the history of laser development that has greatly pushed forward the development of the above-mentioned applied technologies. But, constrained by the mechanism of generation, the pulse width could hardly be further narrowed using the Q modulating technology. In 1964, scientists once again proposed and realized a new mechanism of compressing the pulse width and increasing power, known as the mode locking technology. Owing to its capability of shortening the duration of a pulse
to the order of picosecond (ps, 1 ps = 10^{-12} s), it is also called ultrashort pulse technology. From the 1960s to the 1970s, the ultrashort pulse technology was rapidly developed. By the beginning of the 1980s, Fork and others had proposed the theory of colliding mode locking. More important, they realized colliding mode locking and obtained a stable 90fs optical pulse sequence. By virtue of its capability of generating ultrashort pulses with a pulse width greater than femtosecond (1 fs = 10^{-15} s) and a peak power higher than TW (1 TW = 10^{12} W), the mode locking technology has provided an important means to such disciplines as physics, chemistry, biology, and spectroscopy in learning about the micro-world and ultra-fast process. Owing to the fact that the $Q$ modulating and mode locking technology is capable of making laser radiation highly concentrated in space and time, the monochromatic brightness of laser is enabled to increase by 6 to 9 orders of magnitude over ordinary laser, a new leap in the brightness of the light source. The interaction between lasers of such a high brightness and material has triggered many significant phenomena and novel technologies, making it a powerful instrument in researches in science and technology. If the $Q$ modulating technology is combined with the multilevel amplifying technology, laser of ultra-high power can be generated that can produce extremely high light energy density within an extremely tiny space, thus generating plasmas with a temperature of tens of millions of kilowatt/hours, making it possible to realize the reaction of controllable thermo-nuclear fusion by means of laser ignition.

In addition, for certain laser application fields the laser beam is required to possess very high quality, that is, excellent directionality and monochromaticity. But commonly used lasers often operate in multiple modes (containing the higher-order transverse mode and longitudinal mode), their divergence is rather great, and monochromaticity far from ideal. It’s obvious they are incapable of satisfying the requirements of application in precision interferometry, holography, and fine machining. Therefore, in order to improve the beam quality, the mode selecting technology and frequency stabilizing technology have been investigated and developed, the former consisting in selecting the single mode (fundamental transverse mode and single longitudinal mode) from the modes of laser oscillation. Over the years, a multitude of mode selecting methods have been studied and implemented, the selection of the fundamental transverse mode having greatly improved the divergence of the beam. On the other hand, the selection of the single longitudinal mode makes it possible to obtain the single frequency laser output so as to improve the monochromaticity of laser. However, influenced by all kinds of interference from the outside, lasers can only have rather poor frequency stability. That is, the frequency (wavelength) is randomly fluctuating and hence can hardly be applied in precision measurement. Scientists have for years endeavored to study and develop various frequency stabilizing technologies that can enhance the frequency stability of laser and are of practical value. The essence of the frequency stabilizing technology lies in maintaining the stability of the optical path length in the resonator, that is, having the laser oscillation frequency locked at the standard frequency from beginning to end by means of an electronic servo-control system. In the mid-1960s, the center frequency of the atomic spectral line was chosen as the reference standard. For instance, in 1965, the Lamb dip was used as the reference frequency to perform frequency stabilization for a He-Ne laser. The frequency stability obtained was $10^{-9}$, but the reproducibility was only $10^{-7}$. After 1966, further attempts were made to use the external reference frequency as a standard for frequency stabilization, e.g., the saturated absorption frequency stabilizing method, which consists in using the absorption lines of some molecular gases as the reference frequency. In so doing, the influences of discharge perturbation and pressure broadening could be avoided, thus helping improve the frequency stability. In 1969, making use of CH$_4$ molecules to perform frequency stabilization for the He-Ne laser’s 3.39-μm wavelength, Barger and Hell obtained a frequency stability of $10^{-14}$ and a reproducibility...
of $3 \times 10^{-12}$. Apart from this, iodine is also a frequently used absorption molecule, e.g., $^{127}\text{I}_2$ and $^{129}\text{I}_2$. The frequency stabilization of the 633-μm wavelength for the He-Ne laser, too, yielded very high stability and reproducibility rate. During this decade, there emerged the frequency stabilizing technology for other lasers such as Ar$^+$ and CO$_2$ lasers. The adoption of the laser mode selecting technology and frequency stabilizing technology has made it possible to obtain high quality beams with excellent frequency stability and extremely small angle of divergence. This will not only satisfy the requirements of such applications as precision measurement and holography, but also, with the appearance of advanced frequency stabilizing technology, the standards for length and time frequency have been unified, that is, in the international measuring standards, the international standard that defines the laser wavelength in “meter” can also be used as a standard for the time frequency “second”. Without a doubt, this will exert an extremely far-reaching influence on all fields of physics.

After the laser became commercially available, people immediately began investigating its application in the information technology (the transmission, storage, and processing of information). As laser is a light frequency electromagnetic wave with an extremely high transmission speed and very high frequency and, as a carrier wave, it has a very large content of information capable of providing an excellent information carrier source to applications in such areas as optical communication, optical information processing, etc. Hence the appearance of all kinds of laser modulation technologies one after another. With the unceasing availability of various optical crystal materials, certain physical effects such as the electro-optic, acousto-optic, and magneto-optic effects were successfully utilized to develop a diversity of optical modulation devices and technologies, thereby realizing laser loaded information. In particular, from the end of the 1960s to the beginning of the 1970s, the new conception of the double heterojunction semiconductor laser was put forward by Kressel, Alferov et al., who also succeeded in implementing devices for continuous operation at room temperature. The British scientists of Chinese extraction Gao Kun and Hockham proposed the new concept of light guide fiber based on the principle of total reflection. On the basis of their work, Kapron and others of the American Conning Company successfully developed practical fiber optic products four years later, unraveling the history of vigorous development of the fiber optic communication technology. In addition, in the recent 10-odd years, spatial light modulators have successfully been developed. The very name suggests that they are a kind of device for modulating the distribution of light wave in space. As the devices possess the function of spatially performing real-time modulation of a light beam, they have become the crucial devices in such systems as those of real-time optical information processing, optical computation, optical storage, and optical neural network (ONN), greatly pushing forward the rapid development of the applied technology in those fields.

Prior to the emergence of the laser, the interaction between light and material appeared to be a linear relation while after its appearance, especially following the utilization of the $Q$ modulating and mode locking technology, many highly significant new phenomena and new effects (nonlinear optical effects) ensued, accompanied by the production of a number of nonlinear optical technologies. In 1961, Franken and others observed the second harmonic radiation of ruby laser by focusing the ruby laser beam onto the quartz crystal, which is the phenomenon of frequency doubling. But as the experiment made by Franken and his coworkers was non-phase matched, the conversion efficiency of the second harmonic was very low, about $10^{-8}$. By 1962, Kleinman, Giordmaine, and Maker had put forward their phase matching technology, which consists in using the birefringence effect to achieve phase speed matching, thus realizing effective doubling of frequency. In 1965, the theory of nonlinear optics was approaching perfection day by day, with many important nonlinear optical phenomena occurring one after another, for instance, the generation of photonixing (sum frequency, difference frequency), optical parametric amplification and oscillation, multiple photon absorption, self-focusing, and stimulated scattering. With the development of
the laser technology and nonlinear optical materials, the above-mentioned nonlinear optical phenomena and effects were extensively applied in expanding the laser band (e.g., the laser frequency converter) and changing or controlling the parameters of lasers (e.g., pulse width, power, frequency, stability). Furthermore, the means of investigating the microscopic properties of material (atoms or molecules) was also provided, thus opening up broad vistas of application for laser. A look ahead makes it clear that the effects of the nonlinear interaction between light and material and researches on its application in various nonlinear optical devices will still be one of the important research directions in the years to come, such as the nonlinear effects of optical fiber in fiber optic communication and the formation and transmission of optical solitons. In a word, the reason lasers can show their magic power in so many fields is because use has been made of the combination and operation of different types of lasers and the relevant laser technologies.

The laser technology involves the theoretical knowledge of multiple disciplines and is itself of diversified kinds and daily updated in terms of development. But as far as its fundamentals are concerned, most are implemented based on the utilization of the physical effects induced by the interaction of light with different kinds of material, mainly a discussion on the electro-optic effect, acousto-optic effect, magneto-optic effect, and nonlinear optical effect as well as the adoption of different forms of use in controlling a certain parameter (energy, power, polarization, mode, line width, and pulse width). Despite the different functions of different laser devices and the steady increase in their assortment, the principle is the same, the basic physical laws are invariable. Therefore, as long as these laws and knowledge are grasped, one will be enabled to learn more from what has been learned by inference and solve numerous technical problems. By the 1980s, the laser technology had already developed to the stage of maturity, and its contents including basic theories and techniques, also, had been so enriched that they had become an important component in the development of the disciplines of optoelectronic technology and optoelectronic information. For this reason, it is the indispensable basic knowledge for people engaged in research on optoelectronic technology and its application in different areas. It is the main purpose of this textbook to give a detailed account of the basic concepts and theories, as well as the principal kinds of laser technology, including the modulation technology, Q modulation technology (Q-switching), ultrashort pulse technology, amplifying technology, mode selection technology, frequency stabilizing technology, nonlinear optical technology, and transmission technology and the fundamentals of the role played by the physical effects in technical devices and implementing methods so that the reader will have a fairly clear and systematic understanding of the physical processes of different laser technologies.

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CHAPTER 1
Laser Modulation and Deflection Technology

1.1 The basic concept of modulation

Laser is a kind of light frequency electromagnetic wave with good coherence. Like the radio wave, it can be used as an information transmitting carrier. As laser possesses very high frequency, as high as $10^{13}\sim10^{15}$ Hz, the frequency band available is very wider and hence a great volume of information is transmitted. Furthermore, owing to the extremely short wavelength and extremely rapid transmission speed possessed by laser, what with the independent transmission characteristics of the optic wave, the 2-dimensional information on a plane can be instantly transmitted onto another with very high resolution, providing conditions for 2-D parallel optic information processing. So, laser is an ideal light source for transmitting information, including speech, language, images, and symbols.

If laser is intended to serve as a carrier of information, it is necessary to solve the problem of loading the information onto laser. For instance, for the laser telephone, it is necessary to load the speech information onto laser to let it “carry” the information through a definite transmission path (atmosphere, optical fiber) to the receiver; then it will be identified by the optical receiver and reduced to the original information so as to achieve the goal of communication by telephone. Such a process of loading information onto laser is called modulation and the device by means of which this process is completed is the modulator, while laser is spoken of as carrier and the low frequency information that plays the role of control is called the modulating signal.

The electric field intensity of the laser optical wave is

$$e_c(t) = A_c \cos(\omega_c t + \varphi_c)$$

(1.1-1)

where $A_c$ is amplitude, $\omega_c$ angular frequency, and $\varphi_c$ phase angle. Since laser possesses such parameters as amplitude, frequency, phase, intensity, and polarization, if a certain physical method can be used to change a certain parameter of the optic wave to make it vary in accordance with the law of the modulating signal, then laser is modulated by signal so that the goal of “transporting” information is attained. There are many methods for implementing laser modulation. Depending on the relative relation of the modulator with the laser, they can be divided into external modulation and internal modulation. By internal modulation we mean that the loading of the modulating signal is carried out in the process of oscillation of laser, that is, the modulating signal is used to change the oscillation parameters of the laser so as to change the output characteristics of laser to realize modulation. For instance, the injected semiconductor laser uses the modulation signal to directly change its pump-driven electric current, making the intensity of the laser output modulated (this method is also spoken of as direct modulation). There is another internal modulation method that consists in having the modulation components placed in the laser resonator and using the modulating signal to control the variation of the physical characteristics of the components to change the parameters of the resonator, thereby changing the output characteristics of the laser. The $Q$ modulation technology described in Chapter 2 is in effect a modulation of this type. Internal modulation is currently used mainly in the light source of the injected semiconductor in optical communication. By external modulation we mean placing the modulator on the optical path outside the laser after the formation of laser while changing the physical characteristics of the modulator with the modulation signal. When
laser passes through the modulator, a certain parameter of the optic wave will get modulated. External modulation is easily adjusted and will not affect the laser. In addition, the mode of external modulation is not limited by the operation velocity of the semiconductor devices. Hence its modulation velocity is higher than that of internal modulation, about one order of magnitude higher, and the modulation bandwidth is much wide. Therefore, in the application in high speed large volume optical communication and optical information processing in future, it is bound to capture greater attention.

In terms of the properties modulated, laser modulation can be divided into amplitude modulation, frequency modulation, phase modulation, and intensity modulation, the concept of each of which will be briefly discussed below.

1.1.1 Amplitude modulation

Amplitude modulation means the oscillation of the amplitude of carrier as it varies according to the law of the modulating signal. Suppose the electric field intensity of the laser carrier is as shown in Eq. (1.1-1). If the modulation signal is a cosine function of time, then

\[ a(t) = A_m \cos \omega_m t \]  

(1.1-2)

where \( A_m \) is the amplitude of the modulating signal, \( \omega_m \) the angular frequency of the modulation signal. After the laser amplitude is modulated, the laser amplitude \( A_c \) in Eq. (1.1-1) is no longer a constant, but a function proportional to the modulation signal, the expression of whose amplitude-modulated wave is

\[ e(t) = A_c (1 + m_a \cos \omega_m t) \cos (\omega_c t + \varphi_c) \]  

(1.1-3)

Expanding the above equation with the formula of triangular function, we have the frequency spectrum formula of the AM wave

\[ e(t) = A_c \cos (\omega_c t + \varphi_c) + \frac{m_a A_c}{2} \cos \left( (\omega_c + \omega_m) t + \varphi_c \right) \]

\[ + \frac{m_a A_c}{2} \cos \left( (\omega_c - \omega_m) t + \varphi_c \right) \]  

(1.1-4)

where \( m_a = A_m / A_c \) is called the AM coefficient. It can be seen from Eq. (1.1-4) that the frequency spectrum of the AM wave is composed of three frequency components, of which the first is the carrier frequency component, while the second and third are generated because of the modulation and are called side frequency components, as shown in Fig. 1.1-1. The above analysis is made on the case of the modulation of the single frequency cosine signal. If the modulating signal is a complicated periodic signal, then the frequency spectrum of the AM wave will be made up of the carrier frequency component and two side frequency bands.

1.1.2 Frequency modulation and phase modulation

Frequency modulation and phase modulation are simply the oscillation of frequency or phase of the optical carrier that varies according to the variation law of the modulating signal. As the two kinds of modulated wave are both manifested as a change in the total phase angle \( \varphi(t) \), they are called angular modulation.
1.1 The basic concept of modulation

For frequency modulation, it should be noted that the angular frequency $\omega_c$ in Eq. (1.1-1) is no longer a constant, but varies with the modulating signal, i.e.,

$$\omega(t) = \omega_c + \Delta \omega(t) = \omega_c + k_t a(t) \quad (1.1-5)$$

If the modulating signal is still a cosine function, then the total phase angle of the FM wave is

$$\varphi(t) = \int_0^t \omega(t) \, dt + \varphi_c = \int_0^t [\omega_c + k_t a(t)] \, dt + \varphi_c$$

$$= \omega_c t + \int_0^t k_t a(t) \, dt + \varphi_c \quad (1.1-6)$$

Substituting Eq. (1.1-6) into Eq. (1.1-1), we have the expression of the frequency modulated wave

$$e(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t + \varphi_c) \quad (1.1-7)$$

where $k_f$ is called the proportionality factor and $m_f = \Delta \omega/\omega_m$ is called the frequency modulating factor.

Similarly, phase modulation is simply the variation of phase angle $\varphi_c$ in Eq. (1.1-1) with the variation of the modulating signal; the total phase angle of the phase-modulated wave is

$$\varphi(t) = \omega_c t + \varphi_c + k_p a(t)$$

$$= \omega_c t + \varphi_c + k_p A_m \cos \omega_m t \quad (1.1-8)$$

Then the expression of the phase-modulated wave is

$$e(t) = A_c \cos(\omega_c t + m_p \cos \omega_m t + \varphi_c) \quad (1.1-9)$$

where $m_p = k_p A_m$ is called the phase modulating factor.

Let’s turn to look at the frequency spectrum of the FM and PM wave. Since FM and PM are in the final analysis modulation of the total phase angle, they can be written in a unified form:

$$e(t) = A_c \cos(\omega_c t + m \sin \omega_m t + \varphi_c) \quad (1.1-10)$$

Expanding Eq. (1.1-10) with the triangular formula, we have

$$e(t) = A_c \{ \cos(\omega_c t + \varphi_c) \cos(m \sin \omega_m t)$$

$$- \sin(\omega_c t + \varphi_c) \sin(m \sin \omega_m t) \} \quad (1.1-11)$$

Expand $\cos(m \sin \omega_m t)$ and $\sin(m \sin \omega_m t)$ in the above equation into

$$\cos(m \sin \omega_m t) = J_0(m) + 2 \sum_{n=1}^{\infty} J_{2n}(m) \cos(2n \omega_m t)$$

$$\sin(m \sin \omega_m t) = 2 \sum_{n=1}^{\infty} J_{2n-1}(m) \sin[(2n - 1) \omega_m t]$$

When the modulation factor $m$ is known, the values of all orders of the Bessel functions can be found in the table of Bessel functions. Substituting the above two equations into Eq. (1.1-11) and performing expansion, we have

$$e(t) = A_c \{ J_0(m) \cos(\omega_c t + \varphi_c) + J_1(m) \cos[(\omega_c + \omega_m)t + \varphi_c]$$

$$- J_1(m) \cos[(\omega_c - \omega_m)t + \varphi_c] + J_2(m) \cos[(\omega_c + 2\omega_m)t + \varphi_c]$$

$$+ J_2(m) \cos[(\omega_c - 2\omega_m)t + \varphi_c] + \cdots \}$$

$$= A_c J_0(m) \cos(\omega_c t + \varphi_c) + A_c \sum_{n=1}^{\infty} J_n(m)[\cos \omega_c + n \omega_m t + \varphi_c]$$

$$+ (-1)^n \cos(\omega_c - n \omega_m t + \varphi_c) \quad (1.1-12)$$
Chapter 1 Laser Modulation and Deflection Technology

It can be seen that during single-frequency sinusoidal modulation, the frequency spectrum of the angular modulation wave is made up of the light carrier frequency and the infinitely many pairs of side frequencies symmetrically distributed on its two sides. The frequency interval between the side frequencies is $\omega_m$ and the magnitude of the side frequencies is determined by the Bessel function. If $m = 1$, it will be found from the table of Bessel functions that $J_0(m) = 0.77$, $J_1(m) = 0.44$, $J_2(m) = 0.1$, $J_3(m) = 0.02$, ... The frequency distribution is as shown in Fig. 1.1-2. Obviously, if the modulating signal is not a single-frequency sinusoidal wave, its spectrum will be even more complex. In addition, if the angle modulating factor is rather small, i.e., $m \ll 1$, its frequency spectrum has the same form as that of the AM wave.

1.1.3 Intensity modulation

Intensity modulation is the laser oscillation of the optical carrier wave intensity (light intensity) that varies in obedience to the law of the modulating signal, as shown in Fig. 1.1-3. Usually, the form of intensity modulation is adopted for laser modulation since the receiver (detector) in general directly responds to the variation of the light intensity received.

![Intensity modulation diagram](image)

The light intensity of laser is defined as the square of the electric field of the optical wave, whose expression is

$$I(t) = e^2(t) = A_c^2 \cos^2(\omega_c t + \varphi_c) \quad (1.1-13)$$

Thus, the expression for light intensity of intensity modulation can be written as

$$I(t) = A_c^2 \left[1 + k_p a(t)\right] \cos^2(\omega_c t + \varphi_c) \quad (1.1-14)$$

where $k_p$ is the factor of proportionality. Suppose the modulating signal is a single frequency cosine wave $a(t) = A_m \cos \omega_m t$, which we substitute into Eq. (1.1-14). Letting $k_p A_m = m_p$ (called the intensity modulation factor), we have

$$I(t) = A_c^2 \left[1 + m_p \cos \omega_m t\right] \cos^2(\omega_c t + \varphi_c) \quad (1.1-15)$$

This is a fairly ideal formula for light intensity modulation when the modulation factor $m_p \ll 1$. The frequency spectrum of the light intensity modulating wave can be obtained using a method similar to the one described previously, but the results are slightly different.
1.1 The basic concept of modulation

from those of the spectrum for the modulated wave. With respect to the frequency spectrum distribution, apart from the carrier frequency and the symmetrically distributed side frequencies, there is the low frequency $\omega_m$ as well as the DC component.

In actual application, in order to obtain sufficiently strong anti-interference effects, people often use the secondary modulation mode. That is, first the low frequency signal is used to modulate the frequency of a high frequency subcarrier, then this frequency modulated carrier will be used for intensity modulation, called FM/IM modulation, to cause the intensity of light to vary with the variation of the subcarrier. This is because in the process of transmission, although the atmospheric agitation and other interference wave will be directly superposed onto the optical signal wave, after demodulation, the information will be contained in the frequency modulated subcarrier. So the information will not be interfered with and the original information can be reproduced distortion-free.

1.1.4 Pulse modulation

The modulated waves obtained in the above-mentioned modes of modulation are all continuously oscillating waves, referred to as simulated modulation. In addition, in current optical communication, there are pulse modulation and digital modulation, also spoken of as pulse code modulation, that are extensively adopted for modulation in the discontinuous state. For this kind of modulation, usually electrical modulation (simulated pulse modulation or digital pulse modulation) is performed first, followed by light intensity modulation for the light carrier.

Pulse modulation is a modulating method that is implemented by using an intermittent periodic pulse sequence as the carrier, a certain parameter of which varies in obedience to the law of the modulating signal. That is, first the simulated modulating signal is used to perform electrical modulation for a certain parameter (amplitude, width, frequency, position, etc) of an electric pulse sequence, to make it vary in accordance with the law of the modulating signal, as shown in Fig. 1.1-4, to make it a pulse modulated sequence. Then, this modulated electric pulse sequence should be used to carry out intensity modulation for the light carrier so that the optical pulse sequence of the corresponding variation can be obtained. For instance, if the modulating signal is used to change the time at which each pulse in the electric pulse sequence is generated, then the position of each pulse and that prior to modulation will have a displacement that is proportional to the modulating signal. Such a modulation is called pulse position modulation (PPM), as shown in Fig. 1.1-4(e).

Then, by modulating the light carrier wave emitted by the light source, the corresponding optical pulse position modulated wave can be obtained, whose expression is as follows:

$$e(t) = A_c \cos(\omega_c t + \varphi_c) \quad \text{(When } t_n + \tau_d \leq t \leq t_n + \tau_d + \tau)$$

$$\tau_d = \frac{\tau_p}{2} \left[1 + M(t_n)\right] \quad (1.1-16)$$

where $M(t_n)$ is the amplitude of the modulating signal, and $\tau_d$ is the retardation of the carrier pulse front edge relative to the sampling time $t_n$. To prevent the pulse from getting overlapped onto the period of the adjacent sample, the maximum retardation of the pulse must be smaller than the period of the sample $\tau_p$.

If the modulating signal causes the repeating frequency of the pulse to vary so that the range of frequency shift is proportional to the amplitude of the modulating signal voltage independently of the modulation frequency, then such a modulation is called pulse frequency modulation (PFM). The pulse frequency modulated wave is expressed as

$$e(t) = A_c \cos \left(\omega_c t + \Delta \omega \int M(t_n)dt + \varphi_c\right) \quad \text{(When } t_n \leq t \leq t_n + \tau) \quad (1.1-17)$$
For both pulse position modulation and pulse frequency modulation, a light pulse of very narrow width can be adopted, with the shape of the pulse unchanged but only the position of the pulse or the repeating frequency varying with the variation of the modulating signal. With a fairly strong anti-interference ability, both modulating methods are widely used in optical communication.

![Forms of pulse modulation](image)

**1.1.5 Pulse coding modulation**

This kind of modulation consists in converting the simulating signal into an electric pulse sequence first and then the binary code representing the signal information (PCM digital signal) for intensity modulation of the light carrier prior to performing information transmission.

The implementation of pulse code modulation has to undergo three processes, namely, sampling, quantization and coding.

1. **Sampling**

   By sampling we mean segmenting a continuous signal wave into a discontinuous pulse wave, represented by a pulse sequence of a definite period, and the amplitude of the pulse sequence (called sample value) is in correspondence to the amplitude of the signal wave. That is to say, after sampling, the original simulating signal becomes a pulse amplitude modulated signal. According to the theorem of sampling, if only the sampling frequency is over twofold higher than the highest frequency of the signal transmitted, the waveform of the original signal can be restored.

2. **Quantization**

   By quantization we mean performing level by level round-off treatment of the pulse amplitude modulated wave after sampling by replacing the magnitude of the sampling value with a limited number of representative values. This process is called quantization. So, a sample cannot become a digital signal until it has undergone the process of quantization.
1.2 The electro-optic modulation

3. Coding

The process of transforming a quantized digital signal into the corresponding binary code is called coding; that is, a group of pulses of equal amplitude and equal width are used as “numerals” and “with” pulse and “without” pulse are used to represent the numerals “1” and “0”, respectively. Then the series of electric pulses that reflect the law of the digital signal are added to a modulator to control the output of laser, with the maximum of the laser carrier representing “1” bit of the binary code of information sample amplitude while the zero value of the laser carrier, “0” bit. Thus, different combinations of numerals will be able to represent the information intended to be transmitted. For this form of modulation, both a broader bandwidth and high anti-interference capability are required. So it is widely used in digital fiber communication.

Despite the different kinds of laser modulation, the mechanism of operation of the modulation is invariably based on the diverse physical effects such as the electro-optic effect, acousto-optic effect, and magneto-optic effect. Below we shall have a discussion on each of the basic principles and modulating methods of the electro-optic, acousto-optic, and magneto-optic modulation.

1.2 The electro-optic modulation

Under the action of an applied electric field, the refractive index of certain crystals will vary. When a light wave goes through the medium, its transmission characteristics vary under the influence. Such a phenomenon is referred to as the electro-optic effect, which is widely used to realize control over the light wave (phase, frequency, polarization, and intensity) and developed as a variety of optical modulation devices, optical deflection devices, electro-optic filtering devices, etc.

1.2.1 The physical basis of electro-optic modulation

The law of propagation of light wave in a medium is constrained by the distribution of the medium refractive index which, in turn, is closely related to its medium constant. It has been proved by both theory and experiment that the medium’s dielectric constant is related to the distribution of the electrical charges in the crystal. When an electric field is applied on the crystal, a redistribution of the bound electrical charges will ensue and may lead to a slight deformation of the ionic lattice. The result will be a change in the dielectric constant until the change in the refractive index of the crystal. So the refractive index will become a function of the applied electric field and the crystal’s refractive index can now be expressed by the power series of the applied electric field $E$ as

$$n = n_0 + \gamma E + hE^2 + \cdots$$

(1.2-1)

or

$$\Delta n = n - n_0 = \gamma E + hE^2 + \cdots$$

(1.2-2)

where $\gamma$ and $h$ are constants and $n_0$ is the refractive index before application of the electric field. In Eq. (1.2-2), $\gamma E$ is the primary term, the variation of the refractive index caused by what is known as the linear electro-optic effect or Pockels effect while the variation of the refractive index induced by the secondary term $hE^2$ is called the secondary electro-optic effect or Kerr effect. For most electro-optic crystalline materials, the primary effect is more appreciable than the secondary effect and the secondary term can be neglected (only in a centrosymmetric crystal, owing to the absence of the primary electro-optic effect, will the secondary electro-optic effect become rather obvious). So we shall only discuss the linear electro-optic effect.
1. The electrically induced variation of the index of refraction

There are two methods for analyzing and describing the electro-optic effect: the method using the electromagnetic theory and that using geometric figures. The former involves tedious and complicated mathematical derivation while the latter, owing to its use of the refractive index ellipsoid, also called indicatrix, is intuitive and convenient and is therefore preferred in practice.

When no external electric field is applied on the crystal, the refractive index ellipsoid in the coordinate system of the principal axis is depicted by the following equation:

\[
\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (1.2-3)
\]

where \(x, y, \) and \(z\) represent the directions of the principal axis of the medium, that is to say, within the crystal the electric displacement \(D\) along these directions and electric field intensity are parallel to each other; \(n_x, n_y,\) and \(n_z\) are the main refractive indices of the refractive index ellipsoid. As this equation can be used to depict the characteristics of the light wave propagation in the crystal, it is possible to infer the influence of the crystal on the law of optical wave propagation after an external electric field is applied on the crystal.

An analysis can also be made by the aid of the variation of the factors \(1/n_x^2, 1/n_y^2, 1/n_z^2\) in the equation of the index ellipsoid.

When an electric field is applied on the crystal, “deformation” of the ellipsoid will take place. The corresponding equation of the index ellipsoid will change into the following form:

\[
\left(\frac{1}{n_x^2}\right)_1 x^2 + \left(\frac{1}{n_y^2}\right)_2 y^2 + \left(\frac{1}{n_z^2}\right)_3 z^2 + 2 \left(\frac{1}{n_x^2}\right)_4 yz + 2 \left(\frac{1}{n_y^2}\right)_5 xz + 2 \left(\frac{1}{n_z^2}\right)_6 xy = 1 \quad (1.2-4)
\]

It is known by comparing Eq. (1.2-3) with Eq. (1.2-4) that, owing to the action of the external electric field, the factors \((1/n^2)\) of the index ellipsoid will linearly vary as a result, the quantity of variation being defined as

\[
\Delta \left(\frac{1}{n_i^2}\right)_i = \sum_{j=1}^{3} \gamma_{ij} E_j \quad (1.2-5)
\]

where \(\gamma_{ij}\) is called the linear electro-optic factor, \(i\) takes values \(1 \sim 6,\) and \(j\) takes values \(1\sim3.\) Equation (1.2-5) can be expressed in the matrix form of a tensor as

\[
\begin{bmatrix}
\Delta \left(\frac{1}{n_1^2}\right) \\
\Delta \left(\frac{1}{n_2^2}\right) \\
\Delta \left(\frac{1}{n_3^2}\right) \\
\Delta \left(\frac{1}{n_4^2}\right) \\
\Delta \left(\frac{1}{n_5^2}\right) \\
\Delta \left(\frac{1}{n_6^2}\right)
\end{bmatrix} =
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \gamma_{13} \\
\gamma_{21} & \gamma_{22} & \gamma_{23} \\
\gamma_{31} & \gamma_{32} & \gamma_{33} \\
\gamma_{41} & \gamma_{42} & \gamma_{43} \\
\gamma_{51} & \gamma_{52} & \gamma_{53} \\
\gamma_{61} & \gamma_{62} & \gamma_{63}
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} \quad (1.2-6)
\]
where $E_x$, $E_y$, and $E_z$ are components of the electric field along the $x$, $y$, and $z$ directions; the $6 \times 3$ matrix with elements $\gamma_{ij}$ is called the electro-optic tensor. The value of each element is determined by the specific crystal and is a quantity that characterizes the strength of inductance polarization. Below we shall make an analysis with the frequently used KDP crystal.

KDP (KH$_2$PO$_4$) class crystals belong in the tetragonal system, $42m$ point group, and are negative single axis crystals. Hence there is $n_x = n_y = n_0$, $n_z = n_e$, and $n_0 > n_e$. The electro-optic tensor of such crystals is

$$\begin{bmatrix}
\gamma_{41} & 0 & 0 \\
0 & 0 & 0 \\
0 & \gamma_{52} & 0 \\
0 & 0 & \gamma_{63}
\end{bmatrix}$$

(1.2-7)

Furthermore, $\gamma_{41} = \gamma_{52}$. Hence such crystals have only two independent electro-optic factors, i.e., $\gamma_{41}$ and $\gamma_{63}$. Substitution of Eq. (1.2-7) into Eq. (1.2-6) yields

$$\Delta \left( \frac{1}{n^2} \right)_1 = 0, \quad \Delta \left( \frac{1}{n^2} \right)_4 = \gamma_{41}E_x$$

$$\Delta \left( \frac{1}{n^2} \right)_2 = 0, \quad \Delta \left( \frac{1}{n^2} \right)_5 = \gamma_{41}E_y$$

$$\Delta \left( \frac{1}{n^2} \right)_3 = 0, \quad \Delta \left( \frac{1}{n^2} \right)_6 = \gamma_{63}E_z$$

(1.2-8)

Substituting Eq. (1.2-8) into Eq. (1.2-4), we obtain the equation for the new index ellipsoid:

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2\gamma_{41}yzE_x + 2\gamma_{41}xzE_y + 2\gamma_{63}xyE_z = 1$$

(1.2-9)

It can be seen from Eq. (1.2-9) that the applied electric field has led to the appearance of the “crossed” term in the equation for the index ellipsoid, which shows that, upon application of the electric field, the principal axis of the ellipsoid will no longer be parallel to the $x^-$, $y^-$, and $z$-axes. Therefore, we have to find a new coordinate system, so that Eq. (1.2-9) will become the “principal axis” in this coordinate system since only by so doing will it be possible to determine the effect of the electric field on light propagation. For simplicity, make the direction of the applied electric field parallel to the $z$-axis, that is, $E_z = E$, $E_x = E_y = 0$.

Thus, Eq. (1.2-9) becomes

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2\gamma_{63}xyE_z = 1$$

(1.2-10)

In order to seek a new coordinate system $(x', y', z')$ so that the equation for the ellipsoid will not contain any crossed term, we have the following form:

$$\frac{x'^2}{n_x'^2} + \frac{y'^2}{n_y'^2} + \frac{z'^2}{n_z'^2} = 1$$

(1.2-11)

where $x'$, $y'$, and $z'$ represent the directions of the ellipsoid’s principal axis after application of the electric field. This principal axis is usually called the inductive principal axis; $n_{x'}$, etc.
\(n_{y'}\), and \(n_{z'}\) are the principal refractive indices in the new coordinate system. As \(x\) and \(y\) in Eq. (1.2-10) are symmetrical, the \(x\), \(y\) coordinates can be rotated around the \(z\)-axis an \(\alpha\) angle. Thus the transformation relation from the old coordinate system into the new one is

\[
\begin{align*}
    x &= x' \cos \alpha - y' \sin \alpha \\
    y &= x' \sin \alpha + y' \cos \alpha
\end{align*}
\]  

(1.2-12)

Substituting Eq. (1.2-12) into Eq. (1.2-10), we have

\[
\begin{align*}
    \left[ \frac{1}{n_o^2} + \gamma_{63} E_z \sin 2\alpha \right] x'^2 + \left[ \frac{1}{n_o^2} - \gamma_{63} E_z \sin 2\alpha \right] y'^2 \\
    + \frac{1}{n_e^2} z'^2 + 2 \gamma_{63} E_z \cos 2\alpha x' y' &= 1
\end{align*}
\]  

(1.2-13)

Letting the crossed term be zero, that is, \(\cos 2\alpha = 0\), we find \(\alpha = 45^\circ\) and the equation becomes

\[
\left( \frac{1}{n_o^2} + \gamma_{63} E_z \right) x'^2 + \left( \frac{1}{n_o^2} - \gamma_{63} E_z \right) y'^2 + \frac{1}{n_e^2} z'^2 = 1 \]  

(1.2-14)

This is the new equation of ellipsoid after an electric field is applied on the KDP class crystal along the \(z\)-axis, as shown in Fig. 1.2-1. Comparing Eq. (1.2-14) with Eq. (1.2-11), we have

\[
\begin{align*}
    \frac{1}{n_x'} &= \frac{1}{n_o^2} + \gamma_{63} E_z \\
    \frac{1}{n_y'} &= \frac{1}{n_o^2} - \gamma_{63} E_z \\
    \frac{1}{n_z'} &= \frac{1}{n_e^2}
\end{align*}
\]  

(1.2-15)

As \(\gamma_{63}\) is very small (about \(10^{-10}\) m/V), in general, \(\gamma_{63} E_z \ll \frac{1}{n_o^2}\), using the differential formula \(d \left( \frac{1}{n^2} \right) = -\frac{2}{n^3} dn\), that is, \(dn = -\frac{n^3}{2} d \left( \frac{1}{n^2} \right)\), we have

\[
\begin{align*}
    \Delta n_x &= -\frac{1}{2} n_o^3 \gamma_{63} E_z \\
    \Delta n_y &= \frac{1}{2} n_o^3 \gamma_{63} E_z \\
    \Delta n_z &= 0
\end{align*}
\]  

(1.2-16)

So

\[
\begin{align*}
    n_x' &= n_o - \frac{1}{2} n_o^3 \gamma_{63} E_z \\
    n_y' &= n_o + \frac{1}{2} n_o^3 \gamma_{63} E_z \\
    n_z' &= n_e
\end{align*}
\]  

(1.2-17)

It can be seen that, when an electric field is applied on the KDP crystal along the \(z\)-axis, it is turned from a single-axis crystal into a bi-axial one, the principal axis of the refractive...
1.2 The electro-optic modulation

index ellipsoid having rotated round the \( z \)-axis an angle of 45°. This rotational angle has nothing to do with the magnitude of the applied electric field, but the variation of the index of refraction is proportional to the electric field. The \( \Delta n \) value in Eq. (1.2-16) is called the electrically induced refractive index variation. This is the physical basis of implementing such technologies as electro-optic modulation, \( Q \) modulation, and mode locking using the electro-optic effect.

2. Electro-optic phase retardation

Below we shall make an analysis of how the electro-optic effect causes phase retardation. In actual application, the electro-optic crystal is always cut up along certain particular directions of the relative optical axis. The electric field is also applied on the crystal along the direction of a certain principal axis. There are two frequently used methods; one consists in having the direction of the electric field coincide with the clear direction, called the longitudinal electro-optic effect, and the other, the transverse electro-optic effect, with the direction of the electric field normal to the clear direction. We shall still use the KDP crystal as an example for our analysis. After an electric field is applied on the crystal along its \( z \)-axis, the cross section of its refractive index ellipsoid is as shown in Fig. 1.2-2. If the light wave propagates along the \( z \) direction, then its birefringence characteristics depend on the ellipse formed by the intersection of the ellipsoid with the plane normal to the \( z \)-axis. In Eq. (1.2-14), letting \( z = 0 \), we have the equation of the ellipse,

\[
\left( \frac{1}{n_0^2} + \gamma_{63} E_z \right) x'^2 + \left( \frac{1}{n_0^2} - \gamma_{63} E_z \right) y'^2 = 1 \tag{1.2-18}
\]

One quadrant of the ellipse is shown as the shaded part in the figure, the long and short half-axes of which overlap \( x' \) and \( y' \), respectively, \( x' \) and \( y' \) being the directions of polarization of the two components. The corresponding refraction indices \( n_{x'} \) and \( n_{y'} \) will be determined by Eq. (1.2-17).

When a beam of linearly polarized light is injected into the crystal along the \( z \)-axis direction, it is resolved into two vertical polarized components along the \( x' \) and \( y' \) directions. Owing to the difference in the refractive index between the two, the propagation speed of the light vibrating along the \( x' \) direction is high while that of the light vibrating along the \( y' \) direction is low. After a length \( L \), the light paths they have traveled are \( n_{x'} L' \) and \( n_{y'} L \), respectively. So the phase retardations of the two polarized components are, respectively,

\[
\varphi_{n_{x'}} = \frac{2\pi}{\lambda} n_{x'} L = \frac{2\pi L}{\lambda} \left( n_0 - \frac{1}{2} n_0^3 \gamma_{63} E_z \right)
\]

\[
\varphi_{n_{y'}} = \frac{2\pi}{\lambda} n_{y'} L = \frac{2\pi L}{\lambda} \left( n_0 + \frac{1}{2} n_0^3 \gamma_{63} E_z \right)
\]

Therefore, when the two beams of polarized light penetrate the crystal, there will be a phase difference

\[
\Delta \varphi = \varphi_{n_{y'}} - \varphi_{n_{x'}} = \frac{2\pi}{\lambda} (n_{y'} - n_{x'}) L = \frac{2\pi}{\lambda} L n_0^3 \gamma_{63} E_z = \frac{2\pi}{\lambda} n_0^3 \gamma_{63} V \tag{1.2-19}
\]

It can be seen from the above analysis that this phase retardation is completely induced by the birefringence due to the electro-optic effect and is therefore called the electro-optic phase.
Chapter 1 Laser Modulation and Deflection Technology

retardation. \( V = E_z L \) in the equation is the voltage applied along the \( z \)-axis of the crystal. When the electro-optic crystal and the clear wavelength are determined, the variation of phase difference is only dependent on the applied voltage to vary proportionately.

In Eq. (1.2-19), the voltage applied is called the half-wave voltage when the optical path difference \( (n_x' - n_y') \) between the two vertical components \( E_x' \) and \( E_y' \) is half of a wavelength (the corresponding phase difference being \( \pi \)), usually denoted by \( V_\pi \) or \( V_{\lambda/2} \).

From Eq. (1.2-19), we have

\[
V_{\lambda/2} = \frac{\lambda}{2n_0^3\gamma_{63}} = \frac{\pi c_0}{\omega n_0^3\gamma_{63}} \tag{1.2-20}
\]

The half-wave voltage is an important parameter for characterizing the quality of the performance of an electro-optic crystal. The lower this voltage is, the better it will be, especially in the broad frequency band and high frequency condition. When the half-wave voltage is low, the modulating power needed will be low. The half-wave voltage can usually be measured with the static method by applying DC voltage; then the electro-optic factor of the crystal can be found using Eq. (1.2-20). For this reason, accurate measurement of the half-wave voltage is extremely important for research on electro-optic crystalline materials.

The half-wave voltage and the \( \gamma_{63} \) values of KDP class crystals found with the static method are listed in Tab. 1.2-1.

Tab. 1.2-1 Half-wave voltage and \( \gamma_{63} \) (for \( \lambda = 0.550 \mu m \)) of KDP type (\( \bar{4}2 \) m crystal class) crystals

<table>
<thead>
<tr>
<th>Crystal</th>
<th>Formula</th>
<th>( n_0 )</th>
<th>( V_\lambda/kV )</th>
<th>( \gamma_{63} \times 10^{-10}/(cm/V) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADP</td>
<td>NH₄H₂PO₄</td>
<td>1.526</td>
<td>9.2</td>
<td>8.4</td>
</tr>
<tr>
<td>D-ADP</td>
<td>NH₄D₂PO₄</td>
<td>1.521</td>
<td>6.55</td>
<td>11.9</td>
</tr>
<tr>
<td>KDP</td>
<td>KH₂PO₄</td>
<td>1.512</td>
<td>7.45</td>
<td>10.6</td>
</tr>
<tr>
<td>D-KDP</td>
<td>KD₂PO₄</td>
<td>1.508</td>
<td>3.85</td>
<td>20.8</td>
</tr>
<tr>
<td>RbDP</td>
<td>RbH₂PO₄</td>
<td>1.510</td>
<td>5.15</td>
<td>15.5</td>
</tr>
<tr>
<td>ADA</td>
<td>NH₄H₂AsO₄</td>
<td>1.580</td>
<td>7.20</td>
<td>9.2</td>
</tr>
<tr>
<td>KDA</td>
<td>KH₂AsO₄</td>
<td>1.569</td>
<td>6.50</td>
<td>10.9</td>
</tr>
<tr>
<td>D-KDA</td>
<td>KD₂AsO₄</td>
<td>1.564</td>
<td>3.95</td>
<td>18.2</td>
</tr>
<tr>
<td>RbDA</td>
<td>RbH₂AsO₄</td>
<td>1.562</td>
<td>4.85</td>
<td>14.8</td>
</tr>
<tr>
<td>D-RbDA</td>
<td>RbD₂AsO₄</td>
<td>1.557</td>
<td>3.40</td>
<td>21.4</td>
</tr>
<tr>
<td>CsDA</td>
<td>CsH₂AsO₄</td>
<td>1.572</td>
<td>3.80</td>
<td>18.6</td>
</tr>
<tr>
<td>D-CsDA</td>
<td>CsD₂AsO₄</td>
<td>1.567</td>
<td>1.95</td>
<td>36.6</td>
</tr>
</tbody>
</table>

The half-wave voltage of a crystal is a function of the wavelength. Figure 1.2-3 shows the relationship between some phosphates in \( V_{\lambda/2} \) and wavelength. It can be seen from the figure that within the range measured (400–700 nm), this relationship is linear.

3. Change of the optical polarization state

It is known from the above analysis that the difference between two polarized components in phase speed will cause one component to have a phase difference relative to the other and the action of this phase difference will change the polarization state of the outgoing beam. It is known from “physical optics” that the “wave plate” can be used as a transformer for the light wave polarization state, whose
change of the polarization state of the incoming light is determined by the thickness of the wave plate. In general, the outgoing synthetic vibration is an elliptically polarized light, which is expressed mathematically as

\[
\frac{E_x'^2}{A_1^2} + \frac{E_y'^2}{A_2^2} + \frac{2E_x'E_y'}{A_1A_2}\cos \Delta \varphi = \sin^2 \Delta \varphi
\]

(1.2-21)

Here we adopt a phase retardation crystal that varies proportionately with the applied voltage (equivalent to an adjustable polarization state transformer). Therefore, it is now possible to transform the polarization state of the incoming light wave into the polarization state needed using the electrical method. For illustration, let’s first have a look at the change of polarization under specified conditions.

1. **The electro-optic modulation**

   (1) When no electric field is applied on the crystal, \( \Delta \varphi = 2n\pi \) \((n = 0, 1, 2, \cdots)\), the above equation is reduced to

   \[
   \left(\frac{E_x'}{A_1} - \frac{E_y'}{A_2}\right)^2 = 0
   \]

   or

   \[
   E_y' = \left(A_2/A_1\right)E_x' = E_{x'} \tan \theta
   \]

   (1.2-22)

   This is a rectilinear equation, showing that the synthetic light that has gone through the crystal is still linearly polarized light and its direction coincides with the polarization direction of the incoming light. This is equivalent to the action of “a quarter of wave plate”.

   (2) When an electric field \( (V_{\lambda/4}) \) is applied on the crystal so that \( \Delta \varphi = \left(n + \frac{1}{2}\right)\pi \), Eq. (1.2-21) can be reduced to

   \[
   \frac{E_x'^2}{A_1^2} + \frac{E_y'^2}{A_2^2} = 1
   \]

   (1.2-23)

   This is a positive elliptic equation. When \( A_1 = A_2 \), its synthetic light will become circularly polarized light. This is equivalent to the action of “a quarter of wave plate”.

   (3) When the applied electric field \( (V_{\lambda/2}) \) causes \( \Delta \varphi = (2n + 1)\pi \), Eq. (1.2-21) can be reduced to

   \[
   \left(\frac{E_x'}{A_1} + \frac{E_y'}{A_2}\right)^2 = 0 \quad \text{or} \quad E_y' = -\left(A_2/A_1\right)E_x' = E_{x'} \tan(-\theta)
   \]

   (1.2-24)

   The above equation shows that the synthetic light has become linearly polarized light, but the direction of polarization has rotated a \( 2\theta \) angle relative to the incident light (If \( \theta = 45^\circ \), then it has rotated \( 90^\circ \), along the \( y \) direction). The crystal has played the role of a half wave plate.

   To sum up, suppose a beam of linearly polarized light is injected normal to the \( x'-y' \) plane and vibrates along the direction of the \( x \)-axis, and the moment it enters the crystal \( (z = 0) \) it is resolved into two mutually perpendicular polarized components \( x' \) and \( y' \). After traveling a distance \( L \),

   Component \( x' \) is
   \[
   E_{x'} = Ae^{i\left[w_{x'}t - \left(\frac{\omega_c}{c}\right)(n_0 - \frac{n_0}{2}n_0^3\gamma_{63}E_z)\right]L}
   \]

   Component \( y' \) is
   \[
   E_{y'} = Ae^{i\left[w_{x'}t - \left(\frac{\omega_c}{c}\right)(n_0 + \frac{n_0}{2}n_0^3\gamma_{63}E_z)\right]L}
   \]

   (1.2-25)

   (1.2-26)

   The phase difference between the two components at the outgoing surface of the crystal can be obtained from the difference between the indices in the above two equations:

   \[
   \Delta \varphi = \frac{\omega_c n_0^3\gamma_{63}V}{c}
   \]

   (1.2-27)
Figure 1.2-4 shows the two components $E_x'(z)$ and $E_y'(z)$ at a certain instant (to facilitate observation, the two vertical components are plotted separately) as well as the scanned loci of the apices of the light field vectors at different points along the path. At $z = 0$, the phase difference $\Delta \varphi = 0$, the light field vector is linearly polarized light along the $x$ direction; at point $e$, $\Delta \varphi = \frac{\pi}{2}$, the synthetic light field vector becomes clockwise, rotating circularly polarized light; at point $i$, $\Delta \varphi = \pi$, the synthetic light field vector becomes linearly polarized light along the $y$ direction, having rotated $90^\circ$ relative to the incident polarized light. If a polarizer normal to the direction of the incident light polarization is placed at the output end of the crystal, when the voltage applied on the crystal varies in the range $0 \sim \lambda/2$, the light output from the analyzer is merely a component along the $y$ direction of the elliptically polarized light. Therefore, the variation of the polarization state (polarization modulation) is transformed into one of light intensity (intensity modulation).

1.2.2 electro-optic intensity modulation

There are two cases of implementing electro-optic modulation using the Pockels effect. For one, the electric field applied on the crystal is basically uniform spatially but variable temporally. When a beam of light has gone through the crystal, an electrical signal varying with time can be converted into an optical signal, with the information to be transmitted embodied by the intensity or phase variation of the optical wave. This is mainly applied in such fields as optical communication or optical switches. The other case is one in which the electric field applied on the crystal is adequately distributed spatially, with electric field images formed, that is, distributed with the intensity transmissivity or phase of the variation of the $x$ and $y$ coordinates, but invariable temporally or varying slowly, thereby modulating the light wave passing through it. The spatial light modulator to be described later is a case in point, but for this section we shall discuss the former.

1. longitudinal electro-optic modulation

Figure 1.2-5 shows a typical structure of longitudinal electro-optic intensity modulation. The electro-optic crystal (KDP) is placed between two orthogonal polarizer, of which the polarizing initiating $P_1$’s polarizing direction is parallel to the $x$-axis of the electro-optic crystal while the polarizing direction of the analyzer $P_2$ is parallel to the $y$-axis. After
an electric field is applied along the \( z \)-axis of the crystal, they will rotate 45° to become inductive principal axes \( x' \) and \( y' \). Hence the incident light beam along the \( z \)-axis becomes linearly polarized light parallel to the \( x \)-axis via \( P_1 \) and is resolved into two components along the \( x' \) and \( y' \) directions after entering the crystal \((z = 0)\), their amplitude (equal to \( \frac{1}{\sqrt{2}} \) that of the incident light) and phase being equal and expressed by

\[
E_{x'} = A \cos \omega_c t \\
E_{y'} = A \cos \omega_c t
\]

or expressed by complex numbers as

\[
E_{x'}(0) = A \\
E_{y'}(0) = A
\]

As light intensity is proportional to the square of the electric field, the intensity of the incident light is

\[
I_i \propto |E_{x'}(0)|^2 + |E_{y'}(0)|^2 = 2A^2 \quad (1.2-28)
\]

After light goes through length \( L \) of the crystal, owing to the electro-optic effect, a phase difference \( \Delta \phi \) is generated between the two components \( E_{x'} \) and \( E_{y'} \). Thus,

\[
E_{x'}(L) = A \\
E_{y'}(L) = A \exp(-i\Delta \phi)
\]

So, the total electric field intensity is the sum of the projections of \( E_{x'} \) (\( L \)) and \( E_{y'} \) (\( L \)) in the \( y \) direction, i.e.,

\[
(E_y)_0 = A \sqrt{2} [\exp(-i\Delta \phi) - 1]
\]

Its corresponding output light intensity is

\[
I \propto [(E_y)_0 \cdot (E_y^*)_0] = \frac{A^2}{2} \left[ |\exp(-i\Delta \phi) - 1| |\exp(i\Delta \phi) - 1| \right] = 2A^2 \sin^2 \left( \frac{\Delta \phi}{2} \right) \quad (1.2-29)
\]

Comparing the intensity of the outgoing light with that of the incident light and considering the relationship between Eq. (1.2-9) and Eq. (1.2-20), we have

\[
T = \frac{I}{I_i} = \sin^2 \left( \frac{\Delta \phi}{2} \right) = \sin^2 \left[ \frac{\pi V}{2V_\pi} \right] \quad (1.2-30)
\]
where \( T \) is called the transmissivity of the modulator. Based on the above-mentioned relationship, the curve of the light intensity modulating characteristics can be plotted as shown in Fig. 1.2-6. It can be seen from the figure that, in general, the relationship between the output characteristics of the modulator and the externally applied voltage is nonlinear. If the modulator is operating in the nonlinear part, the modulating light will get distorted. In order to obtain linear modulation, a fixed phase retardation \( \pi/2 \) can be introduced to make the voltage of the modulator biased at the operating point with \( T = 50\% \). There are two frequently used methods. One consists in applying an additional bias voltage \( V_{\lambda/4} \) apart from the signal voltage already applied on the modulating crystal. But this method needs more complicated circuitry and the operating point is not stable. For the other method, a quarter-wave plate should be inserted on the optical path of the modulator, the quick and slow axes forming an angle of 45° with the crystal’s principal axis \( x \), causing a fixed phase difference \( \pi/2 \) to be generated between the two components \( E_x' \) and \( E_y' \). So the total phase difference in Eq. (1.2-30):

\[
\Delta \varphi = \frac{\pi}{2} + \frac{V_m}{V_{\pi}} \sin \omega_m t = \frac{\pi}{2} + \Delta \varphi_m \sin \omega_m t
\]

Expanding \( \Delta \varphi_m \sin \omega_m t \) in the above equation using the Bessel function identity, we have

\[
T = \frac{I}{I_1} = \frac{1}{2} \left[ 1 + \sin(\Delta \varphi_m \sin \omega_m t) \right]
\]

Expanding \( \Delta \varphi_m \sin \omega_m t \) in the above equation using the Bessel function identity, we have

\[
T = \frac{I}{I_1} = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{J_{2n+1}(\Delta \varphi_m)}{J_1(\Delta \varphi_m)} (2n + 1) \omega_m t
\]

It can thus be seen that the output modulating light contains higher order harmonic components that make the modulating light distorted. To obtain linear modulation, it is indispensable to have the higher order harmonic controlled within the allowable range. Suppose the amplitude of the fundamental frequency wave and that of the higher order harmonic are \( I_1 \) and \( I_{2n+1} \), respectively; then the ratio of the higher order harmonic to the fundamental frequency wave constituent is

\[
\frac{I_{2n+1}}{I_1} = \frac{J_{2n+1}(\Delta \varphi_m)}{J_1(\Delta \varphi_m)} (n = 0, 1, 2, \cdots)
\]
1.2 The electro-optic modulation

If we take $\Delta \varphi_m = 1$ rad, then $J_1(1) = 0.44$, $J_3(1) = 0.02$, $I_3/I_1 = 0.045$; that is, the third-order harmonic is 5% of the fundamental wave. Within this range approximately linear modulation can be obtained, so

$$\Delta \varphi_m = \pi \frac{V_m}{V_\pi} \leq 1 \text{ rad} \quad (1.2-34)$$

is taken as the criterion for linear modulation. Now, substitution of $J_1(\Delta \varphi_m) \approx \frac{1}{2} \Delta \varphi_m$ into Eq. (1.2-32) yields

$$T = \frac{I}{I_0} \approx \frac{1}{2} \left[ 1 + \Delta \varphi_m \sin \omega_m t \right] \quad (1.2-35)$$

Hence to obtain linear modulation, the modulating signal is required to be not too big (small signal modulation). Then, the output light intensity modulated wave is just the linear reproduction of the modulating signal $V = V_m \sin \omega_m t$. If the condition of $\Delta \varphi_m \ll 1$ rad cannot be fulfilled (big signal modulation), then the light intensity modulated wave will get distorted.

The above-discussed electro-optic modulator possesses such advantages as simple structure, stable operation, and freedom from the influence of spontaneous birefringence. Its shortcoming is that its half wave voltage is too high, especially when the modulated frequency is rather high, resulting in great power loss.

2. Transverse electro-optic modulation

It has been mentioned that, according to physical optics, there are three different modes of applying the transverse electro-optic effect, namely, ① apply the electric field along the $x$-axis direction, with the light transmitting direction normal to the $z$-axis and forming an included angle of $45^\circ$ with either the $x$- or $y$-axis (the crystal being $45^\circ - z$ cut), ② apply the electric field along the $x$-axis direction (i.e., the direction of the electric field is normal to the optic axis), with the light transmitting direction normal to the $z$-axis and forming an included angle of $45^\circ$ with the $x$-axis (the crystal being $45^\circ - x$ cut), and ③ apply the electric field along the $y$-axis direction, with the light transmitting direction normal to the $y$-axis and forming an included angle of $45^\circ$ with the $z$-axis (the crystal being $45^\circ - y$ cut). Here the analysis will be made with only the first mode of application of the KDP class crystal as an example.

The transverse electro-optic modulation is as shown in Fig. 1.2-7. As the applied electric field is along the $z$-axis, similar to the longitudinal application, $E_x = E_y = 0$ and $E_z = E$. The principal axes $x, y$ rotate $45^\circ$ to $x', y'$, and the three corresponding indices of refraction are as shown in Eq. (1.2-17). But the light transmitting direction is normal to the $z$-axis and is injected along the $y'$ direction (the polarizing direction of the incident light forms an angle of $45^\circ$ with the $z$-axis) and will be resolved into two components vibrating along the $x'$ and $z$ directions after entering the crystal, their refractive indices being $n_{z'}$ and $n_z$, respectively. If the length of the crystal in the light transmitting direction is $L$, the thickness (the distance between two electrodes) is $d$, and the applied voltage $V = E_z d$, then the phase difference between the two outgoing optical waves

$$\Delta \varphi = \frac{2\pi}{\lambda} (n_{z'} - n_z)L = \frac{2\pi}{\lambda} \left[ (n_0 - n_e)L - \frac{1}{2} n_0^3 \gamma_{63} \left( \frac{L}{d} \right) V \right] \quad (1.2-36)$$

It can be seen that the $\gamma_{63}$ transverse electro-optic effect of the KDP crystal causes the phase difference of the light wave after passing the crystal to include two terms. The first term is the phase retardation due to the spontaneous double refraction of the crystal itself unrelated to the applied electric field and has no contribution to the operation of
the modulator. Instead, when the temperature of the crystal varies, it will bring about disadvantageous influences. So effort should be made to eliminate (compensate for) it. The second term is the phase retardation generated by the action of the applied electric field, which is related to the applied voltage \( V \) and the crystal’s dimension \( (L/d) \). If the crystal’s dimension is appropriately chosen, its half-wave voltage can be reduced.

![Fig. 1.2-7 Schematic diagram of transverse electro-optic modulation](image)

The main shortcoming of the transverse electro-optic modulation with the KDP crystal is the presence of the phase retardation induced by natural birefringence. This implies that, in the absence of an applied electric field, there exists a phase difference in the two polarized components resolved from the linearly polarized light that enters the crystal. When the temperature of the crystal varies, owing to the difference in the rate of variation with the temperature between the refractive indices \( n_o \) and \( n_e \), drift will occur with the phase difference between the two light waves. It has been proved by experiment that the rate of variation with the temperature of the difference between the KDP crystal’s two refractive indices is \( \Delta(n_o - n_e) / \Delta T \approx 1.1 \times 10^{-5} \degree C \). Suppose a KDP crystal of a length \( L = 30 \text{ mm} \) is made into a modulator. When it passes through laser of a wavelength of 632.8 nm, the variation of the phase difference induced by the temperature

\[
\Delta \varphi = \frac{2\pi}{\lambda} \Delta nL = \frac{2\pi}{0.6328 \times 10^{-6}} \times 1.1 \times 10^{-5} \times 0.03 = 1.1\pi
\]

If it is required that the variation of phase not exceed 20 mrad, the accuracy of constant temperature of the crystal must be kept within 0.005 \degree C, which is evidently impossible. Therefore, in the KDP crystal transverse modulator, the influence of the natural birefringence will lead to the modulating light getting distorted, and even to the stop of operation of the modulator. So, in actual application, apart from taking as many measures (e.g., heat dissipation, constant temperature, etc.) as possible to reduce the shift of the crystal’s temperature, mainly the structure of a “combined modulator” should be adopted to make the compensation. There are two commonly used methods, one consisting in arranging in series the optical axes of two pieces of crystal of almost completely identical geometric dimensions so that they form an angle of 90° with each other; that is, the \( y’ \)-axis and \( z \)-axis of one piece of crystal are parallel to the \( z \)-axis and \( y’ \)-axis of the other, respectively (Fig. 1.2-8(a)), and the other, having the \( z \)-axis and \( y’ \)-axis of two pieces of crystal arranged parallel to each other in the opposite direction, with a 1/2 wave-plate placed in between (Fig. 1.2-8(b)). The principles of compensation of the two methods are the same. The external electric field is along the direction of the \( z \)-axis (optical axis), but in the two pieces of crystal, the electric field is in the opposite direction relative to the optical axis. When the linearly polarized light is injected into the first piece of crystal along the \( y’ \)-axis, the electrical vector is resolved into light \( e_1 \) along the \( z \) direction and light \( o_1 \) along the \( x’ \) direction. When they pass through the first piece of crystal, the phase difference between two beams of light is

\[
\Delta \varphi_1 = \varphi_{y’} - \varphi_z = \frac{2\pi}{\lambda} \left( n_o - n_e + \frac{1}{2} n_o^3 \gamma_6 \gamma_4 E_z \right) L
\]
1.2 The electro-optic modulation

After the action by the 1/2 wave plate, the directions of polarization of two beams of light each rotate 90° and, after that by the second crystal, the original light $o_1$ has become light $o_2$ while light $e_1$ has become light $e_2$. So after passing through the second piece of crystal, their phase difference is

$$\Delta \varphi_2 = \varphi_z - \varphi_{x'z'} = \frac{2\pi}{\lambda} \left( n_e - n_o + \frac{1}{2} n_{o1263}^3 E_z \right) L$$

Thus, the total phase difference after going through two pieces of crystal is

$$\Delta \varphi = \Delta \varphi_1 + \Delta \varphi_2 = \frac{2\pi}{\lambda} n_{o1263}^3 V \left( \frac{L}{d} \right) \quad (1.2-37)$$

Therefore, if the dimensions and performance of and the influences exerted on two pieces of crystal from the outside are completely identical, then the influences of the natural birefringence can be compensated for. According to Eq. (1.2-37), when $\Delta \varphi = \pi$, the half-wave voltage is $V_{\lambda/2} = \left( \frac{\lambda}{2n_{o1263}^3} \right) \frac{d}{L}$, where what is in the parentheses is the half-wave voltage of the longitudinal electro-optic effect. So

$$\left( V_{\lambda/2} \right)_t = \left( V_{\lambda/2} \right)_l \frac{d}{L}$$

It can be seen that the transverse half-wave voltage is $(d/L)$ times the longitudinal half-wave voltage. Reducing $d$ while increasing the length $L$ can reduce the half-wave voltage. But two pieces of crystal have to be used for this method and so the structure will be complicated. Furthermore, the requirements on their dimension processing are extremely high. For KDP crystals, if the difference in length is 0.1 mm, when the temperature undergoes a change of 1°C the variation of the phase difference will be 0.6°C (for a wavelength of 632.8 nm). So, in general, no one will adopt the transverse mode of modulation for the KDP class crystal. In actual application, as both the GaAs crystal ($n_o = n_e$) of the $43m$ group and the LiNbO$_3$ crystal (electric field applied along the $x$-axis, clear along the $z$ direction) of the $3m$ group are free from the influence of the natural birefringence, the transverse electro-optic modulation is adopted very often.
1.2.3 Electro-optic phase modulation

Shown in Fig. 1.2-9 is a diagram of the principle of electro-optic phase modulation which is composed of an initiating polarizer and a KDP electro-optic crystal. The polarizing axis of the polarizer is parallel to the crystal’s inductive main axis \( x' \) (or \( y' \)) and the electric field is applied along the direction of the \( z \)-axis. Now the polarized light of the incident crystal will no longer be resolved into two components polarizing along \( x' \) and \( y' \), but along either the direction of the \( x' \)- or \( y' \)-axis. So the external electric field does not change the polarization state of the outgoing light, but merely changes its phase, whose variation is

\[
\Delta \varphi_{x'} = \frac{\omega_c}{c} \Delta n_{x'} L \tag{1.2-38}
\]

As the light wave is polarized only along the \( x' \) direction, the corresponding refractive index is \( n_{x'} = n_0 - \frac{1}{2} n_0^3 \gamma_{63} E_z \). If the applied electric field is \( E_{in} = A_c \cos \omega_c t \), then the output light field (at \( z = L \)) becomes \( A_c \cos \left[ \omega_c t - \frac{\omega_c}{c} \left( n_0 - \frac{1}{2} n_0^3 \gamma_{63} E_m \sin \omega_m t \right) L \right] \). By omitting the constant term of the phase angle, which exerts no influence on the modulating effect, the above equation can be rewritten as

\[
E_{out} = A_c \cos(\omega_c t + m \varphi \sin \omega_m t) \tag{1.2-39}
\]

where \( m = \frac{\omega_c n_0^3 \gamma_{63} E_m L}{2c} = \frac{\pi n_0^3 \gamma_{63} E_m L}{\lambda_m} \) is called the phase modulation coefficient.

Expanding the above equation using the Bessel function, we can obtain the form of Eq. (1.1-12).

1.2.4 The electrical performance of the electro-optic modulator

Speaking of the electro-optic modulator, it’s only natural to hope for high modulating efficiency and a modulation bandwidth that meets the requirement. Below we shall make an analysis of the operation characteristics of the electro-optic modulator at different modulating frequencies.

In the foregoing analysis of electro-optic modulation, it is always considered that the frequency of the modulating signal is much lower than that of the light wave (i.e., the wavelength of the modulating signal \( \lambda_m \gg \lambda \)) and \( \lambda_m \) is far greater than the length \( L \) of the crystal. Therefore, within the transit time \( \tau_t = \frac{L}{c/n} \) when the light wave goes through the crystal \( L \), the electric field of the modulating signal is uniformly distributed
1.2 The electro-optic modulation

in all parts of the crystal and the phase retardation obtained by the light wave in all the parts, too, is identical. That is to say, at no moment will the light wave be acted on by a modulating electric field of a different intensity or in the opposite direction. In this case, the modulating crystal with an electrode mounted on it is equivalent to a capacitor; that is, it can be regarded as a lumped element in the circuit, usually spoken of as the lumped parametric modulator, the frequency characteristics of which are mainly influenced by the parameters of the external circuit.

1. Limits to modulated bandwidth by the external circuit

The modulation bandwidth is an important parameter of the light modulator. For the electro-optic modulator, the electro-optic effect itself of the crystal will not limit the frequency characteristics of the modulator as the resonance frequency of the lattice can reach as high as $10^{12}$ Hz. Hence, the modulation bandwidth of the modulator is mainly limited by the parameters of the external circuit.

The equivalent circuit of an electro-optic modulator is as shown in Fig. 1.2-10, in which $V_s$ and $R_s$ represent the modulating voltage and the internal impedance of the modulating source, respectively, $C_e$ is the equivalent capacitance of the modulator, and $R_e$ and $R$ are the resistance of the conductor and the DC resistance of the crystal. It is known from the figure that the actual voltage acting on the crystal is

$$V = \frac{V_s}{R_s + R_e + \frac{1}{(1/R) + i\omega C_0}} = \frac{V_s R}{R_s + R_e + R + i\omega C_0 (R_s R + R_e R)}$$

During modulation at a low frequency, there is in general $R \gg R_s + R_e$, and $i\omega C_0$ is also rather small. Therefore, the signal voltage can be effectively applied on the crystal. But when the modulating frequency is further increased, the impedance of the modulating crystal becomes small. When $R_s > (\omega C_0)$, the greater part of the modulating voltage will drop onto $R_s$, showing that the impedance is not matched between the modulating source and the crystal loaded circuit. Now the modulating efficiency will greatly decrease and, worse still, operation may stop. To realize impedance matching, an inductor $L$ should be connected in parallel at the two ends of the crystal, whose resonance frequency is $\omega_0^2 = (LC_0)^{-1}$, and a shunt resistor $R_L$, whose equivalent circuit is as shown in Fig. 1.2-11. When the frequency of the modulating signal $\omega_m = \omega_0$, the impedance of this circuit will be equal to $R_L$. If we choose $R_L \gg R_e$, then we can make the greater part of the modulating voltage applied onto the crystal. Although this method can increase the efficiency of modulation, the bandwidth of the resonance circuit is limited, whose impedance is rather high only within the range of the frequency internal $\Delta \omega \approx \frac{1}{R_L C_0}$. For this reason, to prevent the modulating wave from becoming distorted, its maximum modulation bandwidth (i.e., the frequency bandwidth occupied by the modulating signal) must be smaller than

$$\Delta f_m = \frac{\Delta \omega}{2\pi} \approx \frac{1}{2\pi R_L C_0} \quad (1.2-40)$$
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As it is, the modulation bandwidth depends on the specific application requirements. Moreover, definite peak phase retardation $\Delta \varphi_m$ is also required, its corresponding driving peak modulating voltage being

$$V_m = \frac{\lambda}{2\pi n_0^3} \Delta \varphi_m$$  \hspace{1cm} (1.2-41)

For the KDP crystal, the driving power needed to obtain the maximum phase retardation $P = V_m^2/2R_L$  \hspace{1cm} (1.2-42)

By making use of Eqs. (1.2-40) and (1.2-41), the above equation can be further written as

$$P = V_m^2 \pi C_0 \Delta f_m = V_m^2 \pi (\varepsilon A/L) \Delta f_m = \frac{\lambda^2 \varepsilon A \Delta \varphi_m^2}{4\pi L n_0^3 \gamma_{63}} \Delta f_m$$ \hspace{1cm} (1.2-43)

where $L$ is the length of the crystal, $A$ is the cross sectional area of normal to $L$, and $\varepsilon$ is the dielectric constant. It is known from the above equation that when the crystal’s category, dimensions, laser wavelength, and phase retardation required are determined, its modulating power will be proportional to its modulating bandwidth.

2. The influence of transition time during high frequency modulation

When the modulating frequency is extremely high, within the transit time the light wave passes the crystal, the electric field may undergo rather significant changes. That is, the modulating voltage in different parts of the crystal is different, especially when the modulation period $(2\pi/\omega_m)$ can be compared to the transition time $\tau_d(= nL/C)$, and the modulating electric field borne by the light wave in various parts of the crystal is different. Hence, the accumulation of the above-mentioned phase retardation will be destroyed. Now the total phase retardation should be obtained from the following integral:

$$\Delta \varphi(L) = \int_0^L aE(t')dz$$ \hspace{1cm} (1.2-44)

where $E(t')$ is the transient electric field, $a = \frac{2\pi}{\lambda n_0^3 \gamma_{63}}$. As the time in which the light wave passes through the crystal is $\tau_d$, and $dz = Cdt'/n$, the above equation can be rewritten as

$$\Delta \varphi(t) = \frac{aC}{n} \int_{t-\tau_d}^t E(t')dt'$$ \hspace{1cm} (1.2-45)

Suppose the applied electric field is a single frequency sinusoidal signal, i.e., $E(t') = A_0 \exp(i\omega_m t')$. Thus,

$$\Delta \varphi(t) = \frac{aC}{n} A_0 \int_{t-\tau_d}^t \exp(i\omega_m t')dt'$$

$$= \Delta \varphi_0 \left[ 1 - \exp\left(-i\omega_m \tau_d \right) \right] \exp(i\omega_m t)$$ \hspace{1cm} (1.2-46)
1.2 The electro-optic modulation

where \( \Delta \varphi_0 = \frac{aC}{n} A_0 \tau_d \) is the peak phase retardation at \( \omega_m \tau_d \ll 1 \); the factor

\[
\gamma = \frac{1 - \exp(-i\omega_m \tau_d)}{i\omega_m \tau_d}
\]

characterizes the decrease of the peak phase retardation induced by the transit time and is thus called the high frequency phase retardation reducing factor. The relationship of \( \gamma \) with \( \omega_m \tau_d \) is as shown in Fig. 1.2-12. Only when \( \omega_m \tau_d \ll 1 \), that is, \( \tau_d \ll \frac{T_m}{2\pi} \), will \( \gamma = 1 \), that is, there is no reducing action. This shows that the transit time of the light wave in the crystal has to be much shorter than the period of the modulating signal to be able to prevent the modulating effect from being affected. This implies that, for the electro-optic modulator, there exists a limit to the highest modulating frequency. For instance, if \( |\gamma| = 0.9 \) is taken as the modulating limit (corresponding to \( \omega_m \tau = \frac{\pi}{2} \)), then the upper limit to the modulating frequency is

\[
f_m = \frac{\omega_m}{2\pi} = \frac{1}{4\tau_d} = \frac{C}{4nL}
\]

For the KDP crystal, if we take \( n = 1.5 \), length \( L = 1 \) cm, then \( f_m = 5 \times 10^9 \) Hz.

3. The traveling wave modulator

In order to be able to operate at higher modulating frequencies without being affected by the transition time, a structural form of the so-called traveling wave modulator can be adopted, as shown in Fig. 1.2-13. Its principle consists in having the modulating signal applied on the crystal in the form of a traveling wave to cause the high frequency modulating field to interact with the light wave field in the form of the traveling wave while enabling the light wave and modulating signal to possess identical phase velocity in the crystal from beginning to end. In this way, the modulating voltage borne by the light wavefront in the process of passing through the entire crystal will be identical so that the effects of the transition time can be eliminated. As the high frequency electric field in most transmission lines is transversely distributed, the traveling wave modulator usually adopts transverse modulation.

Below let’s see why such a structure can realize high frequency modulation and high modulating efficiency. Suppose the wavefront of the light wave enters the incident face \( (z = 0) \) at moment \( t \) and is transmitted to where \( z \) is at moment \( t' \); then we have \( z(t') = \frac{c}{n}(t' - t) \). The phase retardation of the light wave is generated owing to the action
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of the modulating field

\[ \Delta \varphi(t) = a \frac{c}{n} \int_{t}^{t+\tau_d} E[t', z(t')] dt' \]  \hspace{1cm} (1.2-49)

where \( E[t', z(t')] \) is the transient modulating field. If the traveling wave modulating field is taken as

\[ E[t', z(t')] = A_0 \exp[i(\omega_m t' - k_m z)] = A_0 \exp[i(\omega_m t' - k_m (c/n)(t' - t))] \]  \hspace{1cm} (1.2-50)

where \( k_m = \omega_m / c_m \), and \( c_m \) is the phase velocity of the modulating field. Substituting Eq. (1.2-50) into Eq. (1.2-49) and after integration, we have

\[ \Delta \varphi(t) = \Delta \varphi_0 \exp(i\omega_m t) \left\{ \frac{\exp[i\omega_m \tau_d(1 - c/(nc_m))] - 1}{i\omega_m \tau_d(1 - c/(nc_m))} \right\} \]  \hspace{1cm} (1.2-51)

where \( \Delta \varphi_0 = aLA_0 = a(c/n)\tau_d A_0 \) is the phase retardation generated by the DC electric field equal to \( A_0 \). Let

\[ \gamma = \frac{\exp[i\omega_m \tau_d(1 - c/(nc_m))] - 1}{i\omega_m \tau_d(1 - c/(nc_m))} \]  \hspace{1cm} (1.2-52)

be the reduction factor of phase retardation. This is similar to Eq. (1.2-47) of the aforementioned modulator in form. What is different is simply the replacement of \( \tau_d \) here with \( \tau_d[1 - c/(nc_m)] \). It can be seen that, as long as the phase velocity of the light wave in the crystal is equal to that of the high frequency modulating field, i.e., \( c = (nc_m) \), then \( \gamma = 1 \). Now, whatever the length of the crystal, the maximum phase retardation can be obtained. Similar to the treatment of Eq. (1.2-48), the upper limit of the modulating frequency can be obtained as

\[ f_{\text{max}} = \frac{c}{4nL[1 - c/(nc_m)]} \]  \hspace{1cm} (1.2-53)

A comparison of the above equation with Eq. (1.2-8) shows that the modulation with the traveling wave can raise the upper limit to the modulated frequency \( [1 - c/(nc_m)]^{-1} \) times. Currently, the modulation bandwidth of modulators of this type has reached the order of magnitude of dozens of GHz.

1.2.5 The electro-optic waveguide modulator

The various electro-optic modulators discussed in the previous sections are all separate devices of rather large volume and are in general spoken of as “volume modulators”. The limitation common to them is the fact that nearly the entire crystal material is acted on by the externally applied electric field. So a powerful electric field has to be applied on the device to change the optical characteristics of the whole crystal in order that the optical wave it passes through will be modulated. With the development of such applied technologies as
optical communication, optical information processing, and optical computation, there has emerged a brand new discipline at the same time, viz. integrated optics. The principle of integrated optics lies in having the light wave limited to the characteristic of propagating along a definite direction in the waveguide region of the order of magnitude of μm using optical waveguide to enable optical devices to go plane and optical systems to go integrated. To be specific, active devices such as lasers, modulators, and detectors are all “integrated” on the same substrate and connected by means of passive devices such as waveguides and couplers to constitute an integral mini-optical system. Obviously, the medium optical waveguide is the basic component of the integrated optical technology. The waveguide is mainly divided into two kinds, the plane waveguide and rectangular waveguide, and the former, in turn, is subdivided into plate waveguide and gradually changing refractive index waveguide. The plate waveguide is the most frequently used waveguide of the simplest structure as shown in Fig. 1.2-14. It is composed of a thin film waveguide with a high refractive index inserted between the substrate with a low refractive index and the coverage. If the coverage is just air, then \( n_e \approx 1 \). The difference in the refractive index between the thin film waveguide layer and the substrate is in general within the range of \( 10^{-3} \) to \( 10^{-1} \). The thickness of the waveguide layer is in general of the \( \mu m \) order of magnitude (compared with the length of light wave). Total reflection will occur with the light wave on the upper and lower interfaces of the thin film to make the light wave confined in the film layer propagating as a sawtooth-shaped optical path, that is, to the region whose transverse dimension is only of the magnitude of the light wavelength. If signals are input from the outside to control the light wave propagating in the thin film, optical waveguide devices with different functions will be constructed. Such a device whose guide light is controlled from the outside is called the optical waveguide modulator. As it is mainly only acted on by an external electric field in a very small part of the waveguide region to confine the field to the vicinity of the waveguide thin film, the driving power needed by the optical waveguide modulator is reduced by one to two orders of magnitude lower than the volume modulator. A waveguide modulator of the simplest form needs two kinds of material, at least one of which should meet the modulator’s requirements. Furthermore, the two kinds of material have to possess a definite relatively stable refractive index. Not only should a waveguide modulator possess good active properties, but also the material should be capable of combining with another kind of material with a different refractive index to construct a waveguide.

![Fig. 1.2-14 The cross section of a plane waveguide](image)

On the one hand, an optical waveguide modulator constructed from a medium is identical to the volume modulator in terms of the process of control over the optical parameters by its physical (electro-optic, acousto-optic, etc.) effects, that is, the ability to cause the dielectric tensor of the medium to undergo slight variation (i.e., variation of the refractive index) resulting in a phase difference between two propagating modes; and, on the other, it is different from the volume modulator in that, as the action of the external field will lead to the variation of the propagating characteristics of the eigenmode (e.g., the TE mode and...
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TM mode) in the waveguide and the coupling conversion between the two different modes (called the mode coupling modulation), the basic characteristics of the optical waveguide modulator can be depicted using the theory of the medium optical waveguide coupling mode.

1. The principle of modulation of the electro-optic waveguide modulator

The physical basis of realizing modulation by the electro-optic waveguide modulator is the Pockels effect of the crystalline medium. When an electric field is applied on the waveguide, slight variation of the dielectric tensor $\varepsilon$ (refractive index) will be generated, which will bring about the variation of the eigenmode propagation in the waveguide or the transformation of power coupling between different modes. In the coordinate system of the waveguide, there is a one-to-one correspondence relationship between the elements $\Delta \varepsilon_{x,x}$ for the electric field to cause the variation of the medium tensor and the coupling between different modes. If what is contained is only the diagonal dielectric tensor element $\Delta \varepsilon_{x,x}$ or $\Delta \varepsilon_{y,y}$, then self-coupling between the TE modes or TM modes will ensue with only their respective phase varied, thereby generating relative phase retardation, a situation similar to the volume electro-optic phase modulation. However, if in the waveguide coordinate system the variation of the dielectric tensor contains nondiagonal tensor element $\Delta \varepsilon_{x,y}$, then mutual coupling between the TE modes and TM modes will result, leading to the transformation of power between modes. That is, the power of an input mode TE (or TM) will be transformed onto the output mode TM (or TE), whose corresponding coupling equation, after proof by inference, can be reduced to

$$\frac{dA_{m}^{TE}}{dz} = -i\kappa A_{l}^{TM} \exp[-i(\beta_{m}^{TE} - \beta_{l}^{TM})z] \quad (1.2-54a)$$

$$\frac{dA_{l}^{TM}}{dz} = -i\kappa A_{m}^{TE} \exp[i(\beta_{m}^{TE} - \beta_{l}^{TM})z] \quad (1.2-54b)$$

where $A_{m}^{TE}$ and $A_{l}^{TM}$ are the amplitudes of the two modes of the $m$th and $l$th order, respectively, $\beta_{m}^{TE}$ and $\beta_{l}^{TM}$ are the propagation constants of the two modes, respectively, and $\kappa$ represents the mode coupling coefficient, whose expression is

$$\kappa = \frac{\omega}{4} \int_{-\infty}^{\infty} \Delta \varepsilon_{x,y}(x) E_{y}^{(m)}(x) E_{x}^{(l)}(x) dx \quad (1.2-55)$$

Equation (1.2-54) is a description of the coupling between the TE modes and TM modes in the same direction, showing that the variation of amplitude of each mode is a function of the variation of the dielectric tensor (refractive index), the distribution of the mode field, and other mode amplitudes. Suppose the electro-optic material in the waveguide is homogeneous and the distribution of the electric field, too, is homogeneous. When the TE modes and TM modes are completely confined within the waveguide thin film layer and are of the same order ($m = l$), maximum value is taken for the integral of Eq. (1.2-55). Now the field distribution of TE modes and that of TM modes are almost identical, differing only in the direction of the electrical vectors. Moreover, when we have $\beta_{m}^{TE} = \beta_{l}^{TM} = \beta = k_{0}n_{o}$, then the coupling coefficient $\kappa$ will approximately be

$$\kappa = -\frac{1}{2} n_{o}^{2} k_{0} \gamma_{ij} E \quad (1.2-56)$$

Under phase matched conditions, $\beta_{m}^{TE} = \beta_{l}^{TM}$, and the light wave is input in the single mode, $A_{m} = A_{0}$, $A_{l} = 0$. Then the solutions for Eqs. (1.2-54) are

$$A_{m}^{TE}(z) = -iA_{0} \sin \kappa z \quad (1.2-57a)$$

1.2 The electro-optic modulation

\[ A_{TM}(z) = A_0 \cos k z \quad (1.2-57b) \]

It can be seen from Eq. (1.2-56) that, if one wants to obtain complete TE \( \rightarrow \) TM power transformation in the waveguide of length \( L \) \((Z = L)\), it is necessary to satisfy \( kL = \frac{\pi}{2} \).

Now, the length of the optical waveguide is

\[ L = \frac{\pi}{2\kappa} \quad (1.2-58) \]

But when the power is transformed into 0, the corresponding waveguide length is

\[ L = n\pi/\kappa \quad (n = 0, 1, 2, \cdots) \quad (1.2-59) \]

It can be seen that the conditions for this case are the same as those needed by “on” and “off” of the previously described crystal electro-optic modulator. But, in general, the coupling coefficient \( \kappa \) is smaller than the value given in Eq. (1.2-56), so the \( EL \) product needed for achieving complete power transformation should be increased correspondingly.

**Example** There is a GaAs waveguide modulator that takes \( \lambda = 1 \) \( \mu \)m, \( n_0 = 3.5 \), \( n_0^3r = 59 \times 10^{-12} \) m/V, and the externally applied electric field \( E = 10^6 \) V/m. The coupling coefficient \( \kappa = 1.85 \) cm\(^{-1}\) and the length \( L = \frac{\pi}{2\kappa} = 0.85 \) cm needed for power transformation are obtained from Eq. (1.2-56).

The ratio of the output light intensity (TM) to the input light intensity (TE) can be obtained from Eq. (1.2-57):

\[ I/I_0 = \sin^2 \frac{\Delta \varphi}{2} \quad (1.2-60) \]

while

\[ \Delta \varphi = 2\kappa L = \frac{2\pi n_0^3r}{\lambda} \left( \frac{L}{d} \right) V \]

where \( d \) represents the thickness of the waveguide thin film, whose transmissivity curve obtained by experiment is shown in Fig. 1.2-15.

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**Fig. 1.2-15** The transmissivity of a waveguide placed between intersecting polarizers
2. The electro-optic waveguide phase modulation

Figure 1.2-16 shows a schematic diagram of the structure of an electro-optic waveguide phase modulator. With LiNbO$_3$ as the substrate, Ti diffusion forms the plane waveguide and the sputtering method is used to sediment a pair of thin-film electrodes. In the figure, $x, y, z$ represents the waveguide coordinate system and $a, b, c$, the orientation of the crystal axes. When a modulating voltage is applied on the electrodes, if what is propagating in the waveguide is the TM mode, the electric field vector will be along the $z$-axis (corresponding to the $c$-axis of the crystal), the main electric field component being $E_z$. Owing to the variation of the waveguide refractive index due to the electro-optic effect, after the guide wave light passes through the electrode region, its phase varies with the variation of the modulating voltage. That is,

$$\Delta \varphi = \pi n_{0}^{3} \gamma_{33} E_{z} l / \lambda$$

(1.2-61)

where $E_z$ is the electric field component along the direction of the $c$-axis generated by the plane electrode in the crevice, $l$ the length of the electrode, and $\gamma_{33}$ the electro-optic coefficient.

The electro-optic waveguide phase modulation does not involve the mutual coupling among different modes, whose equation of amplitude is

$$\frac{dA_m(x)}{dx} = -i\kappa_{\text{min}} A_m(x)$$

(1.2-62)

whose solution is $A_m(x) = A_m(0) \exp[-i\kappa_{\text{min}} x]$. If the $E_y$ incident wave corresponds to the TM mode, its mode field can be expressed as

$$E_y(x, y, z) = A_y(0) E_y^* (y, z) \exp\{i[\omega t - (\gamma_{yy} + \beta_y) x]\}$$

(1.2-63)

where the self-coupling coefficient is

$$\kappa_{yy} = \frac{\omega^4}{4} \int \int \Delta \varepsilon_{\text{TM-TM}} E_y E_y^* dy dz$$

(1.2-64)

where $\Delta \varepsilon_{\text{TM-TM}}$ is $\Delta \varepsilon_{22}$. In addition, by introducing the expression of the plane waveguide TM mode power normalization

$$\frac{\varepsilon_0 \omega n^2}{2\beta} \int_{-\infty}^{\infty} E_y E_y^* dy = 1$$

which is then substituted into Eq. (1.2-64), the self-coupling coefficient $\kappa_{yy}$ can be determined.

3. The electro-optic waveguide intensity modulation

The device for performing this kind of modulation is similar to the Mach Zehnder (MZ) interferometer. A schematic diagram of the MZ interference type modulator is shown in Fig. 1.2-17. It is composed of the Ti diffusion forking strip-shaped waveguide fabricated by
radio frequency sputtering etching on the substrate of the LiNbO$_3$ crystal as the waveguide base-film. Surface electrodes are fabricated in the middle and on the two sides of the strip-shaped waveguide. This device consists in having two branched beams of light get modulated in the region of the electro-optic effect and meet again prior to their coherent synthesis to realize intensity modulation. For instance, stimulate a TE mode at the input end of the waveguide. Then under the action of the externally applied electric field, the guide mode transmitted in the forked waveguide undergoes phase variation of $\Delta \varphi$ and $-\Delta \varphi$, respectively, owing to the action of the electric field $E_c$ of identical dimensions and opposite signs (since the two branched waveguide structures are completely symmetric). Suppose the length of the electrode is $l$ and the distance between two electrodes is $d$. Then the phase difference between the two modulated waves is $2\Delta \varphi = 2\pi n_{33}^3 E_c l/\lambda$. The coherently synthesized light intensity by the two beams of light at the second confluence of forks will be different as the phase difference differs, thereby resulting in intensity modulation.

In the MZ interferometer-type intensity modulator, in order to increase the modulating depth and reduce insertion loss, it is very important to take the following measures: (1) the branch field angle should not be too big (1° or thereabouts in general), as the greater the field angle is, the greater the radiation loss will be; (2) the waveguide must be designed as single mode to prevent a higher order mode from getting stimulated; (3) the waveguide and electrodes should be rigorously symmetric structurally so that the fixed phase difference between two phase modulated waves will be equal to zero.

For the MZ interferometer-type modulator made of Ti diffusion LiNbO$_3$ waveguide, its modulating depth can reach as deep as 80%, the half-wave voltage is about 3.6 V, power loss 35 $\mu$W/MHz or so, and the modulation bandwidth can reach 17 GHz.

In addition, there are other types of electro-optic waveguide intensity modulators such as the predetermined couple (PC)-type modulator, the electrical absorption (EA)-type modulator, the electro-optic grating modulator, etc. that will not be dealt with here. The reader is referred to books on optical waveguide devices.

### 1.2.6 Electro-optic deflection

The light beam deflection technology is one of the basic technologies in laser application (such as laser display, facsimile, optical storage, etc.). It can be realized by means of the mechanical deflecting mirror, the electro-optic effect, acousto-optic effect, etc. and is divided into two classes according to the different purposes in its application. One is the analog deflection with the deflection angle of light continuously varying, which can depict
the continuous displacement of light, and the other, the discontinuous digit deflection, which consists in making the spatial position of the beam of light “jump vary” in certain specific positions of the chosen space. The former is used in various kinds of display while the latter, mainly in light storage.

1. The principle of electro-optic deflection

The electro-optic deflection is intended to change the direction of propagation of a light beam in space, the principle of which is shown in Fig. 1.2-18. The length of the crystal is $L$, the thickness is $d$, and the beam of light is injected into the crystal along the $y$ direction. If the crystal’s refractive index is a linear function of the coordinate $x$, that is,

$$n(x) = n + \frac{\Delta n}{d} x$$  \hspace{1cm}  (1.2-65)

where $n$ represents the index of refraction at $x = 0$ (below the crystal) and $\Delta n$ the amount of variation of the refractive index on thickness $d$, then the index of refraction at $x = d$ (on the crystal) is $n + \Delta n$. When a plane wave passes through the crystal, the refractive index “experienced” by the light wave’s upper part (ray A) and that by the lower part (ray B) are different, and so are the periods of time needed for passing through the crystal, which are

$$T_A = \frac{L}{c} (n + \Delta n), \quad T_B = \frac{L}{c} (n)$$

respectively. As the time in which one ray passes through the crystal is different from that in which the other passes through, ray A will be a distance behind relative to ray B.

$$\Delta y = \frac{c}{n} (T_A - T_B) = L \frac{\Delta n}{n}$$

This implies that when the light wave reaches the outgoing surface of the crystal, its wave surface has deflected a small angle relative to the axial line of propagation. Its deflection angle (in the output end crystal) is

$$\theta' = -\Delta y \frac{d}{d} = -L \frac{\Delta n}{nd} = -L \frac{dn}{n dx}$$

where $\frac{\Delta n}{d}$ is replaced by the refractive index linear variation rate $\frac{dn}{dx}$. Then, the deflection angle $\theta$ after the beam of light is emitted from the crystal can be found according to the theorem of refraction $\sin \theta / \sin \theta = n$. Suppose $\sin \theta \approx \theta \ll 1$. Then,

$$\theta = \theta' n = -L \frac{\Delta n}{d} = -L \frac{dn}{dx}$$  \hspace{1cm}  (1.2-66)

The minus sign in the equation is introduced by the coordinate system, that is, the turning of angle $\theta$ from $y$ to $x$ means negative. It can be seen from the above discussion that if only
1.2 The electro-optic modulation

The refractive index varies along certain directions when the crystal is under the action of the electric field, then the light beam will be made to deflect when it is injected along a specific direction, the size of its deflection angle being proportional to the linear variation rate of the crystal’s refractive index.

Shown in Fig. 1.2-19 is a dual KDP wedge-shaped prism deflector fabricated on this principle, which is composed of two KDP right angle prisms. The three sides of the prism are along the $x'$, $y'$, and $z$-axis directions, respectively, but the $z$-axes of the two pieces of crystal are parallel to each other in opposite directions while the other two axes have the same orientation.

![Fig. 1.2-19 A dual KDP wedge-shaped prism deflector](image)

The electric field is along the direction of the $z$-axis, while the rays propagate along the $y'$ direction and deflect along the $x'$ direction. In this situation, ray A in the upper part propagates completely in the upper prism and the refractive index “experienced” is

$$n_A = n_0 - \frac{1}{2} n_0^3 \gamma_{63} E_x$$

whereas in the lower prism, as the electric field is opposite in direction with respect to $z$, the refractive index “experienced” by ray B is

$$n_B = n_0 + \frac{1}{2} n_0^3 \gamma_{63} E_x$$

Thus the difference between the upper and lower refractive indices ($\Delta n = n_B - n_A$) is $n_0^3 \gamma_{63} E_z$, which, substituted into Eq. (1.2-66), yields

$$\theta = \frac{L}{d} n_0^3 \gamma_{63} E_x \quad \text{(1.2-67)}$$

For instance, take $L = d = h = 1 \text{ cm}$, $\gamma_{63} = 10.5 \times 10^{-12} \text{ m/V}$, $n_0 = 1.51$, and $V = 1000 \text{ V}$, then $\theta = 35 \times 10^{-7} \text{ rad}$. It can be seen that the electro-optic deflection angle is too small to meet the requirement for use in practice. To make the angle become bigger while keeping the voltage from becoming too high, it is customary to connect a number of KDP prisms in series to construct a deflector of length $mL$, width $d$, and height $h$, as shown in Fig. 1.2-20. The two pieces at the two ends are isosceles triangular prisms each with an angle of $\beta/2$, and those in the middle with the apex angle of $\beta$ are isosceles triangular prisms, their $z$-axes all normal to the face of the plate. The width of a prism is parallel to the $z$-axis and the $z$-axes of two prisms adjacent to each other in tandem are in opposite directions. The electric field is along the direction of the $z$-axis. The directions of the major inductive axes of the prisms are as marked in the figure and the refractive index of the prisms is alternately ($n_0 - \Delta n$) and ($n_0 + \Delta n$), where $\Delta n = \frac{1}{2} n_0^3 \gamma_{63} E_x$, so after a light beam passes through the deflector,
the total deflection angle is \( m \) times the deflection angle of every stage (a pair of prisms), that is,

\[
\theta_t = m\theta = \frac{mLr_0^2\gamma_{63}V}{dh}
\]

(1.2-68)

In general, \( m \) is \( 4 \sim 10 \), \( \theta_t \) but a few minutes. The reason why \( m \) cannot be infinitely increased is because the laser beam has a definite dimension while the magnitude of \( h \) is limited so that it’s impossible for the light beam to deflect beyond \( h \).

2. The electro-optic digital deflection

The electro-optic digital deflection is carried out by combining the electro-optic crystal with the birefringence crystal. The principle of its construction is shown in Fig. 1.2-21. In the figure, \( S \) is the KDP crystal, and \( B \) is the calcite (\( \text{CaCO}_3 \)) or nitrate (\( \text{NaNO}_3 \)) birefringence crystal (separate prism), the deflection being capable of resolving the linearly polarized light into two beams of light that are parallel to each and whose vibrating directions are normal to each other. The interval \( b \) is the degree of splitting, \( \varepsilon \) is the angle of splitting (also called walk-off angle), and \( \gamma \) is the included angle between the normal direction of the incident light and the optical axis. The \( x \)-axis (or \( y \)-axis) of the KDP electro-optic crystal \( S \) should be parallel to the plane formed by the optical axis of the birefringence crystal \( B \) and the crystal face normal. If the polarization direction of a beam of incident light is parallel to the \( x \)-axis in \( S \) (equivalent to o light* for \( B \), when no voltage is applied on \( S \), the polarization state does not change after the light wave passes through \( S \), then the direction still remains unchanged when it passes through \( B \). When a half-wave voltage is applied on \( S \), then the polarized surface of the incident light will rotate 90° and become e light**. We know that light waves along different polarization directions take different orientations of the optical axis, as do the optical paths of their transmission. So, e light now passing through \( B \) has deflected an angle \( \varepsilon \) relative to the incidence direction and e light emitting from \( B \) is \( b \) apart from o light. It is already known from physical optics that when \( n_o \) and \( n_e \) are determined, the corresponding maximum angle of splitting is \( \varepsilon_{\text{max}} = \arctg\left(\frac{n_e^2 - n_o^2}{2n_e n_o}\right) \). Take calcite as an example. Its \( \varepsilon_{\text{max}} = 6^\circ \) (in the wave band of visible light and near-infrared light). A stage one digital polarizer is constructed from the above-mentioned electro-optic crystal and birefringence crystal. The incident linearly polarized light occupies one of two “addresses”

* o light (ordinary light).
** e light (extraordinary light).
1.2 The electro-optic modulation

According to whether a half-wave voltage is or is not applied on the electro-optic crystal, representing state “0” and state “1”, respectively. If \( n \) such digital deflectors are combined, then \( n \)-stage digital deflection can be carried out. Shown in Fig. 1.2-22 is a 3-stage digital deflector as well as how an incident light is resolved into \( 2^3 \) deflection points. The short line (\( \|
\)) on the light path represents the polarization surface is parallel to the surface of paper, “\( .\)”, normal to surface of paper. Of the light emitted the latest, “1” represents that voltage has been applied on a certain electro-optic crystal while “0” represents that no voltage has been applied.

![Fig. 1.2-21 The principle of digital deflection](image1)

![Fig. 1.2-22 A 3-stage digital light deflector](image2)

If it is desired that the controllable positions are distributed in the 2-dimensional direction, it is only necessary to combine two \( n \)-stage deflectors normal to each other. In so doing, \( 2^n \times 2^n \) controllable positions can be obtained.

1.2.7 Problems that should be considered in designing the electro-optic modulator

A high quality electro-optic modulator should mainly satisfy the following requirements: ① the modulator should have a sufficiently broad bandwidth to accomplish highly efficient and distortion-free information transmission; ② little electric power is consumed by the modulator; ③ the linear range of the modulation characteristics curve should be wide; and ④ operation is very stable.

1. Selection of the electro-optic crystal material

The material of the modulation crystal plays the crucial role in the effect of modulation. So, a number of factors should be carefully considered when selecting the crystal material. To begin with, the optical performance should be good, the modulated light wave should be highly transparent and the losses in absorption and dissipation should be small, and the refractive index of the crystal should be uniform. The variation of the refractive index should fulfill \( \Delta n \leq 10^{-4}/\text{cm} \), then the electro-optic coefficient should be great as the half-wave voltage of the modulator and the power consumed are each proportional to \( 1/\gamma^3 \) and \( 1/\gamma^2 \). In addition, the modulation crystal should also have fairly good physicochemical performance.
(mainly the hardness, the threshold of optical destruction, the effects of temperature, air-slate, etc). Table 1.2-2 gives some of the frequently used electro-optic materials.

<table>
<thead>
<tr>
<th>Name of material</th>
<th>Point group</th>
<th>Electro-optic coeff. $\gamma_{ij} \times 10^{-12} \text{m/V}$</th>
<th>Refractive indices $n_e$</th>
<th>$n_0$</th>
<th>Relative dielectric constants $\varepsilon/\varepsilon_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KDP (0.633 µm)</td>
<td>42 m</td>
<td>$\gamma_{41} = 8.6$, $\gamma_{63} = 10.6$</td>
<td>1.47</td>
<td>1.51</td>
<td>$\varepsilon/c = 20$, $\varepsilon_{LC} = 45$</td>
</tr>
<tr>
<td>KD*P (0.633 µm)</td>
<td>42 m</td>
<td>$\gamma_{63} = 23.6$</td>
<td>1.47</td>
<td>1.51</td>
<td>$\varepsilon/c \sim 50$ (24°C)</td>
</tr>
<tr>
<td>ADP (0.633 µm)</td>
<td>42 m</td>
<td>$\gamma_{41} = 28$, $\gamma_{63} = 8.5$</td>
<td>1.48</td>
<td>1.52</td>
<td>$\varepsilon/c = 12$</td>
</tr>
<tr>
<td>Quartz (0.633 µm)</td>
<td>42 m</td>
<td>$\gamma_{41} = 0.2$, $\gamma_{63} = 0.47$</td>
<td>1.55</td>
<td>1.54</td>
<td>$\varepsilon/c = 4.3$, $\varepsilon_{LC} = 4.3$</td>
</tr>
<tr>
<td>CuCl</td>
<td>43 m</td>
<td>$\gamma_{41} = 6.1$</td>
<td>$n_0 = 1.97$</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>ZnS</td>
<td>43 m</td>
<td>$\gamma_{41} = 2.0$</td>
<td>$n_0 = 2.37$</td>
<td>~10</td>
<td></td>
</tr>
<tr>
<td>GaAs (10.6 µm)</td>
<td>43 m</td>
<td>$\gamma_{41} = 1.6$</td>
<td>$n_0 = 3.34$</td>
<td>11.5</td>
<td></td>
</tr>
<tr>
<td>CdTe (10.6 µm)</td>
<td>43 m</td>
<td>$\gamma_{41} = 6.8$</td>
<td>$n_0 = 2.60$</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>LiNbO$_3$ (0.633 µm)</td>
<td>3 m</td>
<td>$\gamma_{33} = 30.8$, $\gamma_{51} = 28$</td>
<td>2.16</td>
<td>2.26</td>
<td>$\varepsilon/c = 50$</td>
</tr>
<tr>
<td>LiTaO$_3$ (30°C)</td>
<td>3 m</td>
<td>$\gamma_{33} = 8.6$, $\gamma_{22} = 3.4$</td>
<td>2.13</td>
<td>2.17</td>
<td>$\varepsilon/c = 43$</td>
</tr>
<tr>
<td>BaTiO$_3$ (30°C)</td>
<td>4 mm</td>
<td>$\gamma_{33} = 23$, $\gamma_{13} = 8.0$</td>
<td>2.365</td>
<td>2.437</td>
<td>$\varepsilon_{LC} = 4300$, $\varepsilon/c = 106$</td>
</tr>
</tbody>
</table>

2. The method of reducing the power loss of the modulator

The half-wave voltage of the KDP class electro-optic crystal is fairly high. To reduce its power loss, it is advisable to adopt the method of connecting n-stage crystals in series (i.e., series connection on the optical path but parallel connection on the circuit). Figure 1.2-23 shows a longitudinal modulating crystal by connecting four KD*P crystals in series with electrodes of identical polarity linked together. In order to make the four crystals all have the same sign with respect to the phase retardation of the two components of the incident polarized light, the $x$-axis and $y$-axis of the crystals are arranged by rotating 90° piece by piece (for example, the $x$-axis and $y$-axis of the second crystal are rotated 90° relative to the $x$-axis and $y$-axis of the first and the third crystals). The result is the addition of the phase retardation, which is equivalent to a reduction in the half-wave voltage. But it is inadvisable to connect too many crystals in series lest the transmissivity should become too low or the capacitance too high.

3. Selection of the modulating voltage

It can be seen from the curve of the modulation characteristics shown in Fig. 1.2-6 that the modulator is already operating at point B. But, if the amplitude of the modulating signal voltage is too high, it will still reach the nonlinear part to cause the distortion of the
modulated light. The relationship between the degree of distortion of the modulated light and the amplitude of the modulating voltage has already been analyzed previously. To make the distortion as little as possible, it is necessary to confine the amplitude of higher-order harmonics within the allowable range. When \( \eta = 100\% \), \( I_{3 \omega}/I_{\omega} \approx 0.05 \), we find \( V \approx 0.383 V_{\lambda/2} \), showing that it can be ensured that no distortion will occur.

4. Selection of the dimensions of the electro-optic crystal

By the dimensions of the electro-optic crystal we mean its length and the size of its cross section. In the longitudinal application of the KDP class crystals, although the half-wave voltage is not related to the length of the crystal, an increase in its length will decrease the capacitance of the modulator (because \( C_0 = \varepsilon A/L \)) to broaden the frequency band. However, the greater the length is, the higher will the requirements be on machining and the accuracy in mounting and modulation. Otherwise, it will be impossible for the optical axis of the crystal to be completely parallel to the direction of light wave propagation. Rather, it will be affected by the spontaneous double refraction of the crystal, thus increasing the instability of phase retardation of the modulator. So \( L \) should not be too long. The size of the transverse cross section mainly depends on the requirements of the light transmitting aperture.

1.3 Acousto-optic modulation

1.3.1 The physical basis of acousto-optic modulation

The acoustic wave is a kind of elastic wave (longitudinal stress wave). When propagating in a medium, it causes the medium to generate corresponding elastic deformation, stimulating the particles in the medium to vibrate along the direction of propagation of the acoustic wave and the density of the medium to alternate between thick and thin. Hence the refractive index of the medium, too, will undergo corresponding periodic variation. This part of the action of the ultrasonic field is like an optical “phase grating”, whose interval (grating constant) is equal to the acoustic wavelength \( \lambda_a \). When the light wave passes through this medium, light diffraction will occur, the intensity, frequency, direction, etc. of the diffracted light varying with the variation of the ultrasonic field.

The propagation of the acoustic wave in a medium is divided into two forms, the form of the traveling wave and that of the standing wave. Shown in Fig. 1.3-1 is the case of the ultrasonic traveling wave in a certain instant, where the dark portions represent that the medium is being contracted and the density becomes thick as is the corresponding index of refraction, while the white portions represent that the medium’s density has become thin. Under the action of the acoustic field of the traveling wave, the refractive index of the medium alternately varies between increase and decrease and pushes forward with \( v_s \) (in general \( 10^3 \) m/s). As the acoustic velocity is only one several hundred thousandth of the velocity of light, for the light wave, the moving “acousto-optic grating” can be regarded as stationary. Suppose the angular frequency of the acoustic wave is \( \omega_s \) and the wave vector is \( k_s \left( \frac{2\pi}{\lambda_a} \right) \). Then the
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Equation of the acoustic wave is

\[ a(x, t) = A \sin(\omega_s t - k_s x) \]  

(1.3-1)

where \( a \) is the transient displacement of the medium particle and \( A \) is the amplitude of the displacement of particles. It can be approximately considered that the variation of the medium’s refractive index is proportional to the rate of variation of the medium particles displacement along the \( x \) direction, i.e.,

\[ \Delta n(x, t) \propto \frac{da}{dx} = -k_s A \cos(\omega_s t - k_s x) \]

or

\[ \Delta n(x, t) = \Delta n \cos(\omega_s t - k_s x) \]  

(1.3-2)

Then the refractive index of the traveling wave is

\[ n(x, t) = n_0 + \Delta n \cos(\omega_s t - k_s x) \]

\[ = n_0 - \frac{1}{2} n_0^3 PS[\cos(\omega_s t - k_s x)] \]  

(1.3-3)

where \( S \) is the strain generated by the medium due to the ultrasonic wave and \( P \) is the elasto-optic coefficient of material.

The acoustic standing wave is composed of two beams of acoustic wave of identical wavelength, amplitude, and phase but opposite directions of propagation, as shown in Fig. 1.3-2. Its equation of acoustic standing wave is

\[ a(x, t) = 2A \cos \left( \frac{2\pi x}{\lambda_s} \sin \left( \frac{2\pi t}{T_s} \right) \right) \]  

(1.3-4)

The above equation shows that the amplitude of the acoustic standing wave is \( 2A \cos \left( \frac{2\pi x}{\lambda_s} \right) \), which is different at different points in the \( x \) direction, but the phase \( \frac{2\pi t}{T_s} \) is the same at all points. Meanwhile, it can also be seen from the above equation that, at the points \( x = 2n\lambda_s/4 \) \( (n = 0, 1, 2, \cdots) \), the amplitude of the standing wave is maximum (equal to \( 2A \)). These points are called the antinodes; the distance between one antinode and another is \( \lambda_s/2 \). At the points \( x = (2n + 1) \lambda_s/4 \), the amplitude of the standing wave is zero. These points are called nodes and the distance between two nodes is also \( \lambda_s/2 \). As the positions of the acoustic standing wave’s antinodes and nodes in the medium are fixed, the grating formed by it is also fixed in space. The variation of the refractive index formed by the acoustic standing wave is

\[ \Delta n(x, t) = 2\Delta n \sin \omega_s t \sin k_s x \]  

(1.3-5)

For the acoustic standing wave, the dense and sparse layers appear twice in the medium and the density remains unchanged at the nodes. Therefore, the refractive index varies once
1.3 Acousto-optic modulation

every half period \((T_s/2)\) at the antinodes, from maximum (or minimum) to minimum (or maximum). In a certain instant during the two times of variation, the indices of refraction in all parts of the medium are identical, being equivalent to a homogeneous medium free from the action of the acoustic field. If the ultrasonic frequency is \(f_s\), then the number of times of appearance and disappearance of grating is \(2f_s\). Thus the modulated frequency of the modulating light obtained after the light wave passes the medium will be twofold of the acoustic frequency.

1.3.2 Two types of interaction between sound and light

Depending on the difference in the height of frequency of the sound wave and the length of action between the sound wave and light wave, the interaction between sound and light can be divided into two types, the Raman-Nath diffraction and Bragg diffraction.

1. Raman-Nath diffraction

When the ultrasonic wave frequency is rather low, the light wave is injected parallel to the surface of the sound wave (i.e., normal to the direction of the sound field propagation), and the interaction length \(L\) of sound and light is rather short, Raman-Nath diffraction will be generated. As the sound velocity is much smaller than that of the light velocity, the acousto-optic medium can be regarded as a stationary plane phase grating. Furthermore, the length of the sound wave \(\lambda_s\) is much greater than the light wavelength \(\lambda\). When the light wave passes through the medium parallelly, it does so almost without going through the sound wave surface. So only its phase is modulated, that is, the light wave surface passing through the optically dense (high refractive index) portion will be delayed while its passage through the optically sparse (low refractive index) portion will be advanced. Thus, there is the phenomenon of convexity and concavity in the passage through the plane wave surface of the acoustic-optic medium, resulting in a creased camber, as shown in Fig. 1.3-3. The secondary wave emitted by the subwave sources on the outgoing wave surface will effect the action of coherence to form multistage light of diffraction that is symmetrically distributed with the incident direction, which is the Raman-Nath diffraction.

Below let’s make a brief analysis on the direction of light wave diffraction and the distribution of light intensity.

Suppose the sound wave in the acousto-optic medium is a plane longitudinal wave (sound column) of width \(L\) and propagating along the \(x\) direction; the wavelength is \(\lambda_s\) (angular frequency \(\omega_s\)), the wave vector \(k_s\) pointing to the \(x\)-axis, and the incident light wave vector \(k_i\) pointing to the direction of the \(y\)-axis, as shown in Fig. 1.3-4. The elastic strain field induced by the sound wave in the medium can be expressed as

\[
S_1 = S_0 \sin(\omega_s t - k_s x)
\]

According to Eq. (1.3-3), there is

\[
\Delta \left( \frac{1}{n^2} \right) = PS_0 \sin(\omega_s t - k_s x)
\]

or

\[
\Delta n = -\frac{1}{2}n^3 PS_0 \sin(\omega_s t - k_s x) \quad (1.3-6)
\]

Then

\[
n(x, t) = n_0 + \Delta n \sin(\omega_s t - k_s x) \quad (1.3-7)
\]
When the acoustic traveling wave is approximately regarded as an ultrasonic field not varying with time, the relation of dependence on time can be neglected. If so, the distribution of the refractive index along the $x$ direction can be reduced to

$$n(x) = n_0 + \Delta n \sin(k_s x) \quad (1.3-8)$$

where $n_0$ is the average refractive index, $\Delta n$ the variation of sound-induced refractive index. As the medium’s refractive index has undergone periodic variation, it is capable of modulating the phase of the incident light wave. If what is investigated is a case of normal incidence by a plane light wave where it is injected into the acousto-optic medium’s front surface at $y = -L/2$ and the incident light wave is

$$E_{in} = A \exp(i\omega_c t) \quad (1.3-9)$$

then the light wave that goes out from $y = L/2$ will no longer be a monochromatic plane wave but a modulated light wave, whose equiphase surface is a creased curved surface determined by the function $n(x)$, whose light field can be written as

$$E_{out} = A \exp[i\omega_c(t - n(x)L/c)] \quad (1.3-10)$$

This outgoing wave front surface can be divided into a number of subwave sources. So at the point P very far away, the total intensity of the diffracted light field is a summation of the contribution of all subwave sources, which is determined by the following integral:

$$E_p = \int_{-q/2}^{q/2} \exp\{ik_i[lx + L\Delta n \sin(k_s x)]\} dx \quad (1.3-11)$$

where $l = \sin \theta$, representing sine of the direction of diffraction, $q$ is the width of the incident beam of light. Substitute $v = 2\pi \Delta n L/\lambda = \Delta nk_i L$ into the above equation and expand to the following form using Euler’s formula:

$$E_p = \int_{-q/2}^{q/2} \{\cos[k_i lx + v \sin(k_s x)] + i \sin[k_i lx + v \sin(k_s x)]\} dx$$
1.3 Acousto-optic modulation

\[
q = \int_{-q/2}^{q/2} \{ \cos(k_l x) \cos[v \sin(k_s x)] - \sin(k_l x) \sin[v \sin(k_s x)] \} dx \\
+ j \int_{-q/2}^{q/2} \{ \sin(k_l x) \cos[v \sin(k_s x)] - \cos(k_l x) \sin[v \sin(k_s x)] \} dx
\]

(1.3-12)

Use the relational expressions

\[
\cos[v \sin(k_s x)] = 2 \sum_{r=0}^{\infty} J_{2r}(v) \cos(2r k_s x) \\
\sin[v \sin(k_s x)] = 2 \sum_{r=0}^{\infty} J_{2r+1}(v) \sin[(2r + 1)k_s x]
\]

where \(J_r(v)\) is the \(r\)th-order Bessel function. By substituting this equation into Eq. (1.3-12) and after integration, the expression of the real part is obtained:

\[
E_p = q \sum_{r=0}^{\infty} J_{2r}(v) \left\{ \frac{\sin((l k_1 + 2r k_s)q/2)}{(l k_1 + 2r k_s)q/2} + \frac{\sin((l k_1 - 2r k_s)q/2)}{(l k_1 - 2r k_s)q/2} \right\} \\
+ q \sum_{r=0}^{\infty} J_{2r+1}(v) \left\{ \frac{\sin[l k_1 + (2r + 1)k_s]q/2}{[l k_1 + (2r + 1)k_s]q/2} - \frac{\sin[l k_1 - (2r + 1)k_s]q/2}{[l k_1 - (2r + 1)k_s]q/2} \right\}
\]

(1.3-13)

while the integral of the imaginary part of Eq. (1.3-12) is zero. It can be seen from Eq. (1.3-13) that the condition for all the terms of the diffracted light field intensity to take the maximum value is

\[
l k_1 \pm mk_s = 0 \quad (m = \text{integer} \geq 0)
\]

(1.3-14)

After angle \(\theta\) and the sound wave vector are determined with one of the terms in them being maximum, the contributions of the other terms are almost equal to zero. Therefore, when \(m\) takes different values, the diffracted light of different \(\theta\) directions takes the maximum value. The azimuth of all levels of diffraction is determined by Eq. (1.3-14):

\[
\sin \theta = \pm m \frac{k_s}{k_1} = \pm m \frac{\lambda}{\Delta \lambda} \quad (m = 0, \pm 1, \pm 2, \cdots)
\]

(1.3-15)

where \(m\) represents the level of the diffracted light, and the intensity of all levels of diffracted light is

\[
I_m \propto J_m^2(v), \quad v = \Delta nk_1 L = \frac{2\pi}{\lambda} \Delta n L
\]

(1.3-16)

To sum up, as a result of Raman-Nath acousto-optic diffraction, the light wave is resolved into a group of diffracted rays of light corresponding to definite diffraction angle \(\theta_m\) (i.e., the direction of propagation) and diffraction intensity, of which the diffraction angle is determined by Eq. (1.3-15) while the diffraction intensity is determined by Eq. (1.3-16), showing that the group of diffracted rays of light is discrete. Since \(J_m^2(v) = J_m^2(v)\), the diffracted light of all levels is symmetrically distributed on the two sides of the zero-level diffracted light and the intensities of the same level are equal, which is one of the main features of Raman-Nath diffraction.

In addition, as

\[
J_0^2(v) + 2 \sum_{1}^{\infty} J_m^2(v) = 1
\]

it can be seen that the sum of the maximum intensities of the diffracted light at all levels without absorption should be equal to the incident light intensity, that is, the optical power.
is conservative. In the above analysis, the factor of time is neglected and the physical image of the Raman-Nath acousto-optic action is obtained using a fairly simple processing method. But, owing to the action of the light wave and sound wave field, every level of diffracted light wave will generate Doppler frequency shift. According to the principle of conservation of energy, there should be
\[ \omega = \omega_i \pm m\omega_s \]  
(1.3-17)
and all levels of diffracted light intensities will be modulated by an angular frequency of \( \sqrt{2} \omega_s \). But as the ultrasonic wave frequency is \( 10^9 \) Hz, while the frequency of the light wave is as high as an order of magnitude of \( 10^{14} \) Hz, the effect of frequency shift can be neglected.

2. The Bragg diffraction

(1) The normal Bragg diffraction in isotropic media

When the frequency of the sound wave is rather high, the length \( L \) of the acousto-optic action is great, and the beam of light is slantingly injected into the sound wave surface at a definite angle, the light wave will penetrate multiple sound wave surfaces in the medium. So the medium possesses the properties of “volume grating”. When the included angle between the incident light and sound wave surface satisfies definite conditions, the different levels of diffracted light in the medium will interfere with one another while the higher-level diffracted light will offset one another, with only the level 0 or level +1 (or level −1) (depending on the direction of the incident light) diffracted light appearing; that is, Bragg diffraction is generated, as shown in Fig. 1.3-5. For this reason, if the parameters can be rationally chosen, the ultrasonic field is sufficiently powerful, and it will be possible to have the energy of the incident light almost completely transferred to the level +1 (or level −1) diffraction extremum, thus enabling the energy of the light beam to be fully utilized. Therefore, acousto-optic devices manufactured using the Bragg diffraction effect are fairly efficient.

![Bragg acousto-optic diffraction](image_url)

Below let’s derive the Bragg equation from the condition for strengthening the interference of the wave. For this purpose, the medium the sound wave passes through can be approximately regarded as many partially reflective partially transmissive mirror surfaces \( \lambda_s \) apart. For the traveling wave ultrasonic field, these mirror surfaces will shift along the \( x \) direction with velocity \( v_s \) (as \( \omega_m \leq \omega_s \), in a certain instant, the ultrasonic field can be regarded as stationary and so having no effect on the distribution of the diffracted light intensity). For the standing wave ultrasonic field, it is completely stationary, as shown in
1.3 Acousto-optic modulation

Fig. 1.3-6. When plane waves 1, 2 are injected into the acoustic wave field at angle $\theta_i$, they are reflected in part at points $B$, $C$, $E$, generating diffracted light $1'$, $2'$, $3'$. The condition for the strengthening of coherence of the diffracted light is that the difference in the light path between them should be integral multiples of the wavelength, or it can be said that they must be in phase. Fig. 1.3-6(a) shows a case of diffraction on one and the same mirror surface, where the condition for the incident light 1, 2 to be in phase with $1'$, $2'$ reflected by them at points $B$, $C$ is that the difference in the optical path $AC$–$BD$ be equal to the integral multiples of the wavelength of the light wave. That is,

$$x(\cos \theta_i - \cos \theta_d) = m\frac{\lambda}{n} \quad (m = 0, \pm 1) \quad (1.3-18)$$

If it is required that all the points on the acoustic wave surface satisfy this condition at the same time, there is no other choice but to make

$$\theta_i = \theta_d \quad (1.3-19)$$

That is, only when the incident angle is equal to the diffraction angle will this be realized. The diffraction on two different mirror surfaces $\lambda_s$ apart is as shown in Fig. 1.3-6(b). The condition for $2'$, $3'$ reflected from points $C$, $E$ to be in phase is that their optical path difference be equal to integral multiples of the light wavelength, i.e.,

$$\lambda_s(\sin \theta_i + \sin \theta_d) = \frac{\lambda}{n} \quad (1.3-20)$$

By taking into account $\theta_i = \theta_d$, it follows that

$$2\lambda_s \sin \theta_B = \frac{\lambda}{n}$$

or

$$\sin \theta_B = \frac{\lambda}{2n\lambda_s} = \frac{\lambda}{2n\nu_s f_s} \quad (1.3-21)$$

where $\theta_i = \theta_d = \theta_B$; $\theta_B$ is called the Bragg angle. It is clear that, only when the incident angle $\theta_i$ is equal to the Bragg angle $\theta_B$, will the light wave diffracted on the acoustic wave surface be in phase so that the condition for coherence strengthening is satisfied and the maximum value of diffraction is obtained. The above equation is called the Bragg equation. For instance, for the acousto-optic diffraction in water, suppose the light wavelength $\lambda = 0.5 \mu$m, $n = 1.33$, the acoustic wave frequency $f_s = 500$ MHz, and acoustic velocity $\nu_s = 1.5 \times 10^3$ m/s. Then $\lambda_s = \frac{\nu_s}{f_s} = 3 \times 10^{-6}$ m. From Eq. (1.3-21) we obtain the Bragg angle $\theta_B \simeq 6 \times 10^{-2} \text{ rad} = 3.4^\circ$.

Below we make a brief analysis of the relationship between the Bragg diffracted light intensity and the characteristics of acousto-optic material and acoustic field intensity. According to inference and proof, when the intensity of the incident light is $I_i$, the expressions
of the zero-th and first level diffraction light intensities can be written as

\[ I_0 = I_i \cos^2 \left( \frac{\nu}{2} \right), \quad I_1 = I_i \sin^2 \left( \frac{\nu}{2} \right) \] (1.3-22)

It is known that \( \nu \) is the additional phase retardation generated when the light wave penetrates an ultrasonic field of length \( L \). \( \nu \) can be represented by the variation \( \Delta n \) of the sound-induced refractive index, i.e.,

\[ \nu = \frac{2\pi}{\lambda} \Delta n L \]

Thus

\[ I_1/I_i = \sin^2 \left[ \frac{1}{2} \left( \frac{2\pi}{\lambda} \Delta n L \right) \right] \] (1.3-23)

Suppose the medium is isotropic. It is known from crystal optics that when light wave and sound wave propagate along certain symmetrical directions, \( \Delta n \) is determined by the elasto-optic coefficient \( P \) of the medium and the elastic strain amplitude \( S \) of the medium under the action of the sound field, i.e.,

\[ \Delta n = -\frac{1}{2} n^3 PS \] (1.3-24)

where \( S \) is related to the ultrasonic driving power \( P_s \) while the ultrasonic power is related to the area of the transducer (\( H \) is the width of the transducer, \( L \) the length of the transducer), the sound velocity \( v_s \), and energy density \( \frac{1}{2} \rho v_s^2 S^2 (\rho \text{ is the density of the medium}) \), i.e.,

\[ P_s = (HL) v_s \left( \frac{1}{2} \rho v_s^2 S^2 \right) = \frac{1}{2} \rho v_s^3 S^2 HL \]

Therefore

\[ S = \sqrt{2P_s/HL\rho v_s^3} \]

Thus

\[ \Delta n = -\frac{1}{2} n^3 P \sqrt{\frac{2P_s}{HL\rho v_s^3}} = -\frac{1}{2} n^3 P \sqrt{\frac{2I_s}{\rho v_s^3}} \] (1.3-25)

where \( I_s = P_s/HL \) is called the ultrasonic intensity. Substitution of Eq. (1.3-25) into Eq. (1.3-23) yields

\[ \eta_s = \frac{I_1}{I_i} = \sin^2 \left[ \frac{\pi L}{\sqrt{2\lambda}} \sqrt{\left( \frac{n^6 P^2}{\rho v_s^6} \right) I_s} \right] = \sin^2 \left( \frac{\pi L}{\sqrt{2\lambda}} \sqrt{M_2 I_s} \right) \] (1.3-26)

or

\[ \eta_s = \frac{I_1}{I_i} = \sin^2 \left[ \frac{\pi}{\sqrt{2\lambda}} \sqrt{\left( \frac{L}{H} \right) M_2 P_s} \right] \] (1.3-27)

where \( M_2 = n^6 P^2/\rho v_s^6 \) is a combination of the physical parameters of the acousto-optic medium, a quantity determined by the properties of the medium itself and which is called the quality factor (or the acousto-optic high quality index) of the acousto-optic material. It is one of the principal indices of selecting the acousto-optic medium. It can be seen from Eq. (1.3-27) that: ① If under the condition that the ultrasonic power \( P_s \) is definite, it
is desired that the intensity of the diffracted light should be as high as possible, then it is required that a material of great $M_2$ be selected and the transducer be made long and narrow (i.e., great in $L$, small in $H$); 2) When the ultrasonic power $P_s$ is sufficiently high, such that

$$\frac{\pi}{\sqrt{2\lambda}} \left( \frac{L}{H} \right)^{M_2} P_s$$

is enabled to reach $\frac{\pi}{2}$, $I_1/I_i = 100\%$; and 3) When $P_s$ varies, $I_1/I_i$ will also vary with it, hence by controlling $P_s$ (i.e., controlling the electric power applied on the transducer), the goal of controlling the intensity of the diffracted light can be attained, thus realizing acousto-optic modulation.

(2) The particle model for Bragg acousto-optic diffraction

What is dealt with above is the principle of Bragg interaction between acoustic and light from the point of view of coherence and superposition of the light wave, but the condition for acousto-optic Bragg diffraction can also be obtained from the quantum characteristics of light and acoustic. A light beam can be regarded as a photon (particle) flow of energy $\hbar \omega_i$ and momentum $\hbar k_i$, of which $\omega_i$ and $k_i$ are the angular frequency and wave vector of the light wave. Similarly, an acoustic wave can also be regarded as a phonon flow of energy $\hbar \omega_s$ and momentum $\hbar k_s$. The interaction between acoustic and light can be regarded as a series of collisions between photons and phonons. Every time a collision occurs there will be the annihilation of an incident photon ($\omega_i$) and a phonon ($\omega_s$) while a new (diffracted) photon of frequency $\omega_d = \omega_i + \omega_s$ will be generated at the same time. The flow of these new diffracted photons propagates along the direction of diffraction. According to the principle of conservation of momentum before and after collision, there should be

$$\hbar k_i - \hbar k_s = \hbar k_d$$

that is,

$$k_i \pm k_s = k_d \tag{1.3-28}$$

Similarly, according to the conservation of energy, there should be

$$\hbar \omega_i - \hbar \omega_s = \hbar \omega_d$$

That is,

$$\omega_i \pm \omega_s = \omega_d \tag{1.3-29}$$

where “+” represents absorbing the phonon, “-” releasing the phonon, depending on the relative direction of $k_i$ and $k_s$ during the collision between photon and phonon. That is, the diffracted photon is generated by the photon that has disappeared and the phonon that has been absorbed during the collision, which takes the sign “+” in the formulae and the frequency is $\omega_d = \omega_i + \omega_s$. If in the collision the disappearance of an incident photon leads to the generation of phonon and diffracted photon at the same time, then the “-” sign will be taken, the frequency being $\omega_d = \omega_i - \omega_s$. As the light wave frequency ($\omega_i$) is far higher than the acoustic wave frequency ($\omega_s$), according to Eq. (1.3-29), it can be approximately considered that

$$\omega_d = \omega_i \pm \omega_s \approx \omega_i \tag{1.3-30}$$

Therefore

$$k_d = k_i \tag{1.3-31}$$

So the vector graph of Bragg diffraction is an isosceles triangle, as shown in Fig. 1.3-7. From the figure, $k_i \sin \theta_i + k_d \sin \theta_d = 2k_s \sin \theta_B = k_s$ can be directly derived. Then there is

$$\sin \theta_B = \frac{k_s}{2k_i} = \frac{\lambda}{2n\lambda_s} \tag{1.3-32}$$
\[ \theta_i = \theta_d = \theta_B \]

This is the Bragg equation obtained previously.

(3) Abnormal Bragg diffraction

The above discussion is made under the condition that the wave vectors of the incident light and diffracted light are equal (i.e., \(|k_i| = |k_d|\)); that is, it is assumed that the direction of polarization of the incident light and that of the diffracted light are the same. Therefore, the corresponding refractive indices are equal \((n_i = n_d)\). These properties can be fulfilled only in isotropic media (glass, liquid, and crystals of the cubic system). If the acousto-optic media are anisotropic crystals, the refractive index of the light beam is in general related to the direction of propagation. As the diffracted light propagates along a direction different from that of the incident light, the polarized state of the incident light is different from that of the diffracted light, and the indices of refraction corresponding to them are not equal \((n_i \neq n_d)\), so \(|k_i| \neq |k_d|\). Such Bragg diffraction that occurs in anisotropic media is called abnormal Bragg diffraction.

The abnormal Bragg diffraction no longer has the condition for \(n_i \neq n_d\), and the corresponding geometric relationship is more complicated than that of the normal Bragg diffraction. Its momentum triangular closure condition \(k_d = k_i \pm k_s\) is shown in Fig. 1.3-8 (corresponding to the case of taking “+” in the formulae), where the modes of \(k_i\), \(k_d\), and \(k_s\) are

\[
\begin{align*}
k_i &= 2\pi n_i(\theta_i)/\lambda \\
k_d &= 2\pi n_d(\theta_d)/\lambda \\
k_s &= 2\pi/\lambda_s = 2\pi f_s(\theta)
\end{align*}
\]

(1.3-33)

respectively. From Fig. 1.3-8, according to the law of cosine, we have

\[
\begin{align*}
k_d^2 &= k_i^2 + k_s^2 - 2k_i k_s \cos \left(\frac{\pi}{2} - \theta_i\right) = k_i^2 + k_s^2 - 2k_i k_s \sin \theta_i \\
k_i^2 &= k_d^2 + k_s^2 - 2k_d k_s \sin \theta_d
\end{align*}
\]

from which \(\sin \theta_i\) and \(\sin \theta_d\) can be solved. Then substitution of Eq. (1.3-33) into the above equation yields

\[
\begin{align*}
\sin \theta_i &= \frac{\lambda}{2n_i(\theta_i)\nu_s} \left\{ f_s + \frac{\nu_s^2}{\lambda^2 f_s} [n_i^2(\theta_i) - n_d^2(\theta_d)] \right\} \\
\sin \theta_d &= \frac{\lambda}{2n_d(\theta_d)\nu_s} \left\{ f_s - \frac{\nu_s^2}{\lambda^2 f_s} [n_i^2(\theta_i) - n_d^2(\theta_d)] \right\}
\end{align*}
\]

(1.3-34)

The above equation is called Dixon’s equation, in which \(n_i\) and \(n_d\) are the functions of angles \(\theta_i\) and \(\theta_d\). Therefore, only after the functional relationship of variation of \(n_i\) and \(n_d\) with the angle is determined for a definite medium will it be possible to solve the relationship between \(\theta_i - f_s\) and \(\theta_d - f_s\) using Dixon’s equation, thereby determining the geometric relationship.
1.3 Acousto-optic modulation

of abnormal Bragg diffraction. An analysis of Eq. (1.3-34) provides us with the following characteristics:

1) The first term $\frac{\lambda}{2n_{i}s} f_{s}$ in the equation is exactly the condition for normal Bragg diffraction while the second term exists only in an anisotropic medium, which differs from crystal to crystal. When $f_{s} = f_{0} = \frac{\nu_{s}}{2} \sqrt{n_{d0}^{2} - n_{i0}^{2}}$, $\theta_{i}$ reaches maximum, and $\theta_{d} = 0$, where $f_{0}$ is called the maximum frequency of abnormal Bragg diffraction, $n_{i0} = n_{i}(\theta_{0})$, $n_{d0} = n_{d}(\theta_{0})$, while $\theta_{0}$ and $\theta_{d0}$ are the $\theta_{i}$ and $\theta_{d}$ values corresponding to $f_{0}$. When $f_{s} = f_{0}$, the two terms on the right side of the above equation are equal but when $f_{s} \geq f_{0}$, the second term on the right side is small and can be neglected. Now Dixon’s equation can be reduced to

$$\sin \theta_{i} = \sin \theta_{d} = \frac{\lambda}{2n_{i}s} f_{s}$$  \hspace{1cm} (1.3-35)

That is, it has become a normal Bragg diffraction equation. But when $f_{s}$ approaches or is smaller than $f_{0}$, then it will have a geometric relationship totally different from that of the normal Bragg diffraction.

2) If the two sub-equations of Eq. (1.3-34) are added up, then the second term on the right side of the equation can be eliminated (the difference between $n_{i}$ and $n_{d}$ in the denominator neglected). Then we have

$$\sin \theta_{i} + \sin \theta_{d} \cong \frac{\lambda}{2n_{i}s} f_{s}$$

If it is further considered that if both $\theta_{i}$ and $\theta_{d}$ are very small, then there is

$$\alpha = \theta_{i} + \theta_{d} \cong \frac{\lambda}{2n_{i}s} f_{s}$$  \hspace{1cm} (1.3-36)

This shows that, although the $\theta_{i} - f_{s}$ relationship is different from the normal Bragg diffraction relationship, the relationship between $\alpha$ and $f_{s}$ is completely identical to that of normal Bragg diffraction.

3) As the abnormal Bragg diffraction $|k_{i}| \neq |k_{d}|$, there can exist interaction in the same direction, that is, $k_{i}$, $k_{d}$, and $k_{s}$ are all in the same direction. Obviously, the closure condition of the momentum triangle can be reduced to the scalar form, i.e., $k_{d} = k_{i} \pm k_{s}$. If we substitute into it $k_{i} = \frac{2\pi}{\lambda} n_{i}$, $k_{d} = \frac{2\pi}{\lambda} n_{d}$, and $k_{s} = \frac{2\pi}{\lambda_{s}} = \frac{\pi}{\nu_{s}} f_{s}$, then we can obtain

$$\lambda = \pm \nu_{s}(n_{d} - n_{i})/f_{s}$$  \hspace{1cm} (1.3-37)

This shows that for a definite acousto-optic medium and direction of propagation, the numerator part to the right of the equation is a constant. Now, when white light (or light with complex spectral components) is injected, for a certain definite acoustic frequency $f_{s}$, only the definite wavelength $\lambda$ that satisfies Eq. (1.3-37) can be diffracted. If $f_{s}$ is changed, so should the corresponding diffracted wavelength. By making use of this characteristic, tunable acousto-optic filters can be made.

3. The quantitative standard for distinguishing between Raman-Nath diffraction and Bragg diffraction

In theory, the Raman-Nath diffraction and Bragg diffraction are two extreme cases that appear when varying the acousto-optic diffractive parameters. The main parameters that influence the appearance of the two cases of diffraction are the wavelength of acoustic $\lambda_{s}$, the incident angle $\theta_{i}$ of the light beam, and the length of action $L$ of acoustic and light. In order
to give the quantitative standard for distinguishing between the two kinds of diffraction, the parameter \( G \) is introduced for characterization.

\[
G = k_s^2 L/k_i \cos \theta_i = 2\pi \lambda L/\lambda_s^2 \cos \theta_i
\] (1.3-38)

When \( L \) is small and \( k_s \) is great (\( G \ll 1 \)), the diffraction is Raman-Nath diffraction whereas if \( L \) is great and \( \lambda_s \) is small (\( G \gg 1 \)), we have Bragg diffraction. In order to seek a practical standard, that is, when the parameter \( G \) is so great that, with the exception of the intensities of level 0 and level 1 diffraction light, all the remaining diffracted light intensities are very low and can be neglected. If such is the case, then it can be considered that the region of Bragg diffraction is entered. After years of practice, the following quantitative standard is now universally adopted:

\[
\begin{cases}
G \geq 4\pi & \text{(region of Bragg diffraction)} \\
G > \pi & \text{(region of Raman-Nath diffraction)}
\end{cases}
\] (1.3-39)

To facilitate application, another quantity is introduced:

\[
L_0 = \lambda_s^2 \cos \theta_i/\lambda \approx \lambda_s^2/\lambda
\]

(1.3-40)

Therefore, the above quantitative standard can be written as

\[
\begin{cases}
L \geq 2L_0 & \text{(region of Bragg diffraction)} \\
L \leq \frac{1}{2}L_0 & \text{(region of Raman-Nath diffraction)}
\end{cases}
\] (1.3-41)

where \( L_0 \) is called the characteristic length of acousto-optic devices. The introduction of parameter \( L_0 \) will greatly simplify the design of devices. As \( \lambda_s = v_s/f_s \), \( L_0 \) is related not only to the properties (\( v_s \) and \( n \)) of the medium, but also to the operating conditions (\( f_s \) and \( \lambda_0 \)). In fact, \( L_0 \) is a representation of the main characteristics of the interaction between sound and light.

4. A theoretical analysis of the Bragg diffraction—the theory of the coupled wave

The interaction between sound and light can be regarded as a process of interaction between parameters, that is, first, because of the perturbation by the acoustic field, the electric susceptibility in the medium is made to vary periodically in time and space, making the incident light wave and the acoustic wave in the medium coupled to generate a series of polarized wave with composite frequency, whose angular frequency and wave vector component are, respectively,

\[
\begin{align*}
\omega_d &= \omega_i \pm m\omega_s & (m = \pm 1, \pm 2, \cdots) \\
k_d &= k_i \pm mk_s & (m = \pm 1, \pm 2, \cdots)
\end{align*}
\] (1.3-42)

The re-radiation by these polarized waves of the secondary waves with the above-mentioned composite frequency causes the strengthening of the radiation coherence of the corresponding secondary waves to form the diffracted light at different levels. The variation of the refractive index induced by the elastic wave in the medium is

\[
\Delta n(x, t) = \Delta n \cos(\omega_s t - k_s x)
\] (1.3-43)

The interaction of such a variation with the light field of \( \omega_i \) and \( \omega_d \) will cause additional electric polarization in the medium:

\[
\Delta P(x, t) = 2\sqrt{\varepsilon \varepsilon_0} \Delta n(x, t) E(x, t)
\] (1.3-44)
1.3 Acousto-optic modulation

where \( E(x, t) \) is the sum of the electric field with frequency \( \omega_i \) and \( \omega_d \); the medium polarization formed by \( \Delta n E \) causes the energy exchange between the two light waves \( \omega_i \) and \( \omega_d \). In order to set up the coupling equation of the interaction between acousto and light, we proceed from the fluctuation equation

\[
\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \frac{\partial^2 E}{\partial t^2} + \frac{\mu}{2} \frac{\partial^2 E}{\partial t^2} P^{(NL)}
\]  

(1.3-45)

Suppose the medium is loss-free. Then \( \sigma = 0 \). Equation (1.3-45) is the basic equation of the parametric interaction, that is, the equation of fluctuation of the light wave propagating in a medium in which the refractive index varies. It satisfies both the light field with frequency \( \omega_i \) and that with \( \omega_d \). Suppose what is involved in the incident light field and diffracted light field is both linearly polarized light. For the incident light field with frequency \( \omega_i \), Eq. (1.3-45) can be written as

\[
\nabla^2 E_i = \mu \frac{\partial^2 E_i}{\partial t^2} + \mu \frac{\partial^2 (\Delta P)_i}{\partial t^2}
\]  

(1.3-46)

where \((\Delta P)_i\) is the component of the electric polarization in the direction parallel to \( E_i \), whose frequency is \( \omega_i \). The components of the other frequencies are not in step with \( E_i \), so the average value of their contributions to \( E_i \) is zero. Thus the general electric field of medium polarization is the sum of the following two traveling waves:

\[
E_i(x, t) = \frac{1}{2} E_i(x_i) \exp[j(\omega_i t - k_i x)] + c.c
\]

(1.3-47)

\[
E_d(x, t) = \frac{1}{2} E_d(x_d) \exp[j(\omega_d t - k_d x)] + c.c
\]

where \( k_i \) and \( k_d \) are parallel to the propagation direction of the incident light and that of the diffracted light, respectively. Performing differentiation twice with respect to Eq. (1.3-47) gives

\[
\nabla^2 E_i(x, t) = -k_i^2 E_i + 2j k_i \frac{d E_i}{d x_i} + \nabla^2 E_i \exp[j(\omega_i t - k_i x)]
\]

Suppose \( E_i(x_i) \) varies so slowly that \( \nabla^2 E_i \ll k_i d E_i/d x_i \). Combining the above equation with Eq. (1.3-46) and using the relation \( k_i = \omega_i \sqrt{\mu \epsilon_i} \) gives

\[
k_i \frac{d E_i}{d x_i} = j \mu \left[ \frac{\partial^2}{\partial t^2} (\Delta P_i) \right] \exp[j(\omega_i t - k_i x)]
\]

(1.3-48)

Using the relation \( \Delta P = 2 \sqrt{\varepsilon_0 \Delta n} \Delta n E_i(x, t) \) \( [E_i(x, t) + E_d(x, t)] \) gives

\[
(\Delta P)_i = \frac{1}{2} \sqrt{\varepsilon_0 \Delta n} E_i \exp[j(\omega_s + \omega_d) t - (k_s + k_d) x] + c.c
\]

(1.3-49)

Substitution of Eq. (1.3-49) into Eq. (1.3-48) yields

\[
\frac{d E_i}{d x_i} = -j \eta_i E_d \exp[-j(k_i - k_d - k_s) x]
\]

Similarly, there is

\[
\frac{d E_d}{d x_d} = -j \eta_d E_i \exp[-j(k_i - k_d - k_s) x]
\]

(1.3-50)

where \( \eta_{i,d} = \frac{1}{2} \sqrt{\mu \varepsilon_0} \Delta n = \frac{\omega_i \Delta n}{2c} = \frac{\pi v_i^3}{2\lambda} P S \), which is referred to as the coupling coefficient. It characterizes the intensity of the interaction between acousto and light.
Equation (1.3-50) shows that, under the action of a definite acoustic field, the condition for obtaining maximum energy coupling between the incident light field \( E_i \) and diffracted light field \( E_d \) is

\[ k_i = k_s + k_d \quad (1.3-51) \]

If this Bragg condition is satisfied and, since the acoustic frequency \( \omega_s \) is much smaller than the light frequency \( \omega_i, \omega_d \), by taking \( \eta_i = \eta_d = \eta \), then Eq. (1.3-50) becomes

\[ \frac{dE_i}{dx_i} = -j\eta E_d \\
\frac{dE_d}{dx_d} = -j\eta E_i \quad (1.3-52) \]

In order to solve the above equation, transform the two coordinates \( x_i, x_d \) onto a new coordinate \( \xi \), as shown in Fig. 1.3-9. As it is the angle resolving line along directions \( k_i \) and \( k_d \), we have

\[ x_i = \xi \cos \theta \\
x_d = \xi \cos \theta \]

So Eq. (1.3-52) becomes

\[ \frac{dE_i}{d\xi} = \frac{dE_i}{dx_i} \cos \theta = -j\eta E_d \cos \theta \\
\frac{dE_d}{d\xi} = \frac{dE_d}{dx_d} \cos \theta = -j\eta E_i \cos \theta \quad (1.3-53) \]

The solution for the above equation is

\[ E_i(\xi) = E_i(0) \cos(\eta \xi \cos \theta) - jE_d(0) \sin(\eta \xi \cos \theta) \]
\[ E_d(\xi) = E_d(0) \cos(\eta \xi \cos \theta) - jE_i(0) \sin(\eta \xi \cos \theta) \]

By using the relationship between \( \xi \) and \( x_i, x_d \), the above equation can be written as

\[ E_i(x_i) = E_i(0) \cos(\eta x_i) - jE_d(0) \sin(\eta x_i) \]
\[ E_d(x_d) = E_d(0) \cos(\eta x_d) - jE_i(0) \sin(\eta x_d) \quad (1.3-54) \]

Equation (1.3-54) is exactly the equation that describes the two beams of coupled wave in an isotropic medium that satisfy the Bragg condition. When the amplitude of the incident light is \( E_i(0) \), the frequency is \( \omega_i \), and \( E_d(0) = 0 \), Eq. (1.3-54) becomes

\[ E_i(x_i) = E_i(0) \cos(\eta x_i) \]
\[ E_d(x_d) = -jE_i(0) \sin(\eta x_d) \quad (1.3-55) \]

and there is

\[ |E_i(x_i)|^2 + |E_d(x_d)|^2 = |E_i(0)|^2 \quad (1.3-56) \]

The above equation shows that, for the two kinds of light wave, the optical power is conservative, in the process of interaction between acoustic and light. From the afore-mentioned coupling coefficient \( \eta_{i,d} = \frac{\omega_{i,d} \Delta n}{2c} = \frac{1}{2} \left( \frac{2\pi}{\lambda} \Delta n \right) \) and considering \( x_i = x_d = L / \cos \theta_B \),
### 1.3 Acousto-optic modulation

we define a new quantity \( v = \frac{2\pi \Delta n L}{\lambda \cos \theta_B} \). Therefore, represented by the light intensity, Eq. (1.3-55) can be written as

\[
I_i = I_i(0) \cos^2 \left(\frac{v}{2}\right) \\
I_d = I_i(0) \sin^2 \left(\frac{v}{2}\right)
\]  

(1.3-57)

The variation of the optical power with the distance of action is as shown in Fig. 1.3-10. The acousto-optic diffraction efficiency is defined as the ratio of the output diffracted light intensity to the input light intensity, i.e.,

\[
\eta_s = \frac{I_d(L)}{I_i(0)} = \sin^2 \left(\frac{v}{2}\right)
\]

(1.3-58)

It can be seen from Eq. (1.3-57) that, when \( v/2 = \pi/2 \), \( I_i = 0 \), while when \( I_d = I_i(0) \), all the energy of the incident light will be transferred into the beam of diffracted light; that is, the ideal Bragg diffraction efficiency can reach 100%. That’s why the Bragg diffraction effect is adopted so often in acousto-optic devices.

#### 1.3.3 The acousto-optic volume modulator

1. The composition of the acousto-optic modulator

The acousto-optic modulator is composed of the acousto-optic medium, the electro-acoustic transducer, the acousto absorbing (or reflection) device, and the driving power source, as shown in Fig. 1.3-11.

(1) The acousto-optic medium. The acousto-optic medium is the site of interaction between sound and light. When a beam of light passes through a variable ultrasonic field, because of the interaction between light and ultrasonic field, its outgoing light possesses all
levels of diffracted light that varies with time. Making use of the property of the variation of the intensity of diffracted light with the variation of the intensity of the ultrasonic wave, we shall be able to make the light intensity modulator.

(2) The electro-acoustic transducer (also referred to as the ultrasonic generator). By the use of the anti-piezoelectric effect of certain piezoelectric crystals (quartz, LiNbO$_3$, etc.) or piezoelectric semiconductors (CdS, ZnO, etc.) and under the action of the externally applied electric field, it generates mechanical vibration while forming the ultrasonic wave. So it plays the role of transforming the modulated electric power into sound power.

(3) The acoustic absorbing (or reflection) device. It is placed opposite the source of ultrasonic to absorb the acoustic wave (operating in the traveling wave state) that has already passed through the medium to prevent the generation of disturbance due to a return to the medium. But in order to make the ultrasonic field operate in the standing wave state, it is necessary to replace the acoustic absorbing device with an acoustic reflecting device.

(4) The driving power source. It is used to generate the modulating electric signal to be applied onto the electrodes at the two ends of the electro-acoustic transducer to drive the acousto-optic modulator (transducer) into operation.

2. The operation principle of acousto-optic modulation

Acousto-optic modulation is a physical process of loading information onto the optical frequency carrier wave by means of the acousto-optic effect. The modulating signal is transformed from acting on the electro-acoustic transducer in the form of an (amplitude modulating) electric signal into an ultrasonic field varying in the form of an electric signal. When the light wave passes through the acousto-optic medium, because of the acousto-optic action, the light carrier wave becomes an intensity-modulated wave upon getting modulated.

It is known from the afore-going analysis that, no matter whether it is Raman-Nath diffraction or Bragg diffraction, their diffraction efficiency is related to the additional phase retardation factor $\nu = \frac{2\pi}{\lambda} \Delta n L$, where the refractive index difference $\Delta n$ of the sound wave is proportional to the elastic strain amplitude $S$, while $S \propto$ the sound power $P_s$. So, when the sound wave field following the modulation by the signal makes the sound wave amplitude vary accordingly, the diffracted light intensity, too, will experience corresponding variation with it. The Bragg acousto-optic modulation characteristic curve is similar to that of the electro-optic intensity modulation, as shown in Fig. 1.3-12. It can be seen from the figure that the diffraction efficiency $\eta$ and the ultrasonic power $P_s$ are in the form of nonlinear modulation curve. In order to prevent the modulation from getting distorted, it is necessary to employ an additional ultrasonic offset to enable it to operate in a region of fairly good linearity.

For the Raman-Nath-type diffraction, the operational acoustic frequency is lower than 10 MHz. The operating principle of this kind of modulator is shown in Fig. 1.3-13(a). All levels of its diffracted light intensity are proportional to $J_n^2(\nu)$. If a certain level of diffracted light is taken as the output, the other levels of diffracted light can be shielded with a diaphragm. Then the light beam going out of the aperture is a modulating light varying with $\nu$. As the efficiency of the Raman-Nath diffraction is low, the availability of light energy is also low. The length $L$ of mutual action as specified in the criterion Eq. (1.3-38) is small. When the operating frequency is rather high, its maximum allowable length is too small and the sound power required is very high. Hence the Raman-Nath-type acousto-optic modulator is only limited to operation at low frequencies and has only limited bandwidths.

For the Bragg diffraction, its diffraction efficiency is given by Eq. (1.3-58) as

$$\eta_b = \frac{I_d}{I_i} = \sin^2 \left( \frac{\nu}{2} \right)$$
1.3 Acousto-optic modulation

The principle of operation of the Bragg-type acousto-optic modulator is as shown in Fig. 1.3-13(b). Under the condition of rather low sound power $P_s$ (or sound intensity $I_s$), the diffraction efficiency $\eta_s$ monotonically increases with $I_s$ (appearing to be a linear relation), i.e.,

$$\eta_s \approx \frac{\pi^2 L^2}{2 \lambda^2 \cos^2 \theta_B} M_2 I_s \quad (1.3-59)$$

where the factor $\cos \theta_B$ has taken into account the influence of the Bragg angle on the acousto-optic action. It can be seen from this equation that, if the acousto intensity is modulated, the intensity of the diffracted light will also get modulated. The Bragg diffraction must make the diffracted light beam injected at the Bragg angle $\theta_B$. Meanwhile, only when receiving
the diffracted light beam along the symmetric direction relative to the sound wave surface, will a satisfactory result be obtained. For its high efficiency and fairly broad modulating bandwidth, the Bragg diffraction is frequently adopted.

3. The modulating bandwidth

As an important parameter of the acousto-optic modulator, the modulating bandwidth is a technical index for judging whether information can be transmitted distortion-free. It is subject to the restriction of the Bragg bandwidth. For the Bragg acousto-optic modulator, under the ideal plane light wave and acoustic wave condition, the wave vector is deterministic. Hence for light wave with a given incident angle and wavelength, only a sound wave with a definite frequency and wave vector can fulfill the Bragg condition. When the finite divergent light beam and acoustic wave field are adopted, the finite angle of the wave beam will expand. Therefore, only within a finite acoustic frequency range is the Bragg diffraction allowed to be generated. According to the Bragg diffraction eq. (1.3-21), the relation between the permissible acoustic frequency bandwidth $\Delta f_s$ and the likely quantity of variation of the Bragg angle is obtained:

$$\Delta f_s = \frac{2n\nu_s \cos \theta_B}{\lambda} \Delta \theta_B \quad (1.3-60)$$

where $\Delta \theta_B$ is the quantity of variation of the incident angle and diffraction angle induced by the divergence of the light beam and acoustic beam, or the quantity of variation allowed by the Bragg angle. Suppose the angle of divergence of the incident light beam is $\delta \theta_i$, and that of acoustic wave beam is $\delta \phi$. For a diffraction-restrained wave beam, the relationship between these beam divergence angles with the wavelength and that with the beam width are approximated respectively as

$$\delta \theta_i \approx \frac{2\lambda}{\pi nw_0}, \quad \delta \phi \approx \frac{\lambda_s}{L} \quad (1.3-61)$$

where $w_0$ is the beam waist radius of the incident light beam, $n$ the refractive index of the medium, and $L$ the width of the acoustic beam. Obviously, the range covered by the incidence angle (the included angle between the light wave vector $k_i$ and the acoustic wave vector $k_s$) is

$$\Delta \theta = \delta \theta_i + \delta \phi \quad (1.3-62)$$

If the incident (divergent) light beam propagating inside angle $\delta \theta_i$ is resolved into a number of plane waves along different directions (i.e., different wave vectors $k_i$), for the component of the light beam in a particular direction, there is an acoustic wave with a suitable frequency and wave vector that satisfies the Bragg condition, while the acoustic wave beam contains many Fourier frequency spectral components of an acoustic carrier wave of the central frequency because of the modulation by the signal. Therefore, for every acoustic frequency, the acoustic wave component with many wave vectors in different directions can induce the diffraction of the light wave. Thus, the incident light corresponding to every definite angle has a beam of diffracted light with a divergence angle $2\delta \phi$, as shown in Fig. 1.3-14, while each direction of diffraction corresponds to a different frequency shift. So, in order to resume the modulation of the intensity of the diffracted light beam, it is necessary to make the component of the diffracted light with different frequency shifts mixing in the square-law detector. So it is required that two beams of diffracted light on the farthest boundary (e.g., OA’ and OB’ in the figure) overlap to a certain extent. This requires that $\delta \phi \approx \delta \theta_i$. If we take $\delta \phi \approx \delta \theta_i = \frac{\lambda}{\pi nw_0}$, then the modulated bandwidth is obtained from Eq. (1.3-60) and Eq. (1.3-62):
1.3 Acousto-optic modulation

\[(\Delta f)_m = \frac{1}{2} \Delta f_s = \frac{2nv_s}{\pi w_0} \cos \theta_B \quad (1.3-63)\]

![Diagram of Bragg diffraction with wave beam divergence](image)

The above equation shows that the bandwidth of the acousto-optic modulator is inversely proportional to the transition time \(\frac{w_0}{v_s}\) at which the sound wave passes through the light beam and that a broad modulated bandwidth can be obtained with a light beam of small width. But the divergence angle of the light beam should not be too big, or the level-0 and level-1 diffracted light beams will partially overlap and the modulating effect will be reduced. So it is required that \(\delta \theta_i < \theta_B\). Thus, we can find from Eq. (1.3-21), Eq. (1.3-60), and \(\Delta \theta_i = \theta_B\) that

\[\frac{\Delta f_m}{f_s} \approx \frac{\Delta f}{2f} \leq \frac{1}{2} \quad (1.3-64)\]

That is, the maximum modulated bandwidth \(\Delta f_m\) is approximately equal to half of the sound frequency \(f_s\). Therefore, a large modulated bandwidth can be obtained only by adopting high frequency Bragg diffraction.

4. The diffraction efficiency of the acousto-optic modulator

Another important parameter of the acousto-optic modulator is the diffraction efficiency. According to Eq. (1.3-26), the acoustic intensity needed for obtaining 100% modulation is

\[I_s = \frac{\lambda^2 \cos^2 \theta_B}{2M^2 L^2} \quad (1.3-65)\]

If represented as the sound power needed, it is

\[P_s = HLI_s = \frac{\lambda^2 \cos^2 \theta_B}{2M^2 \left(\frac{H}{L}\right)} \quad (1.3-66)\]

It can be seen that, the greater the quality factor \(M^2\) of the acousto-optic material, the lower the sound power needed for obtaining a 100% diffraction efficiency. Moreover, the cross section of the electro-acoustic transducer should be made long (L long) and narrow (H small), but although an increase in the action length L is helpful to raising the efficiency of diffraction, it will lead to a decrease in the modulated bandwidth (as the acoustic beam
divergence angle $\delta \phi$ is inversely proportional to $L$. A small $\delta \phi$ implies a small modulated bandwidth.

Letting $\delta \phi = \frac{\lambda_s}{2L}$ and using Eqs. (1.3-60)~(1.3-62), we can write the bandwidth as

$$\Delta f = \frac{2n\nu_s\lambda_s}{\lambda L} \cos \theta_B$$  \hspace{1cm} (1.3-67)

Solving $L$ using the above formula and substituting it into the previously given Eq. (1.3-26), we have

$$2\eta_s \Delta f f_0 = \left( \frac{n^7 P^2}{\rho \nu_s^2} \right) \frac{2\pi^2}{X^3 \cos \theta_B} \left( \frac{P_s}{H} \right)$$  \hspace{1cm} (1.3-68)

where $f_0$ is the acoustic center frequency ($f_0 = \nu_s / \lambda_s$). We introduce the factor

$$M_1 = \frac{n^7 P^2}{\rho \nu_s^2} = (n^7 \nu_s^2)M_2$$  \hspace{1cm} (1.3-69)

$M_1$ is the quality factor that characterizes the modulated bandwidth characteristics of the acousto-optic material. The greater the $M_1$ value of an acousto-optic material of which a modulator is made, the greater the modulated bandwidth allowed will be.

In addition, for certain acousto-optic devices (such as the acousto-optic deflector to be discussed below), another factor should be considered; that is, during acousto-optic action, as the incident light beam has a definite width, and the acoustic wave is propagating in the medium with a limited velocity, the acoustic wave needs definite transition time to pass through the light beam, that is, $\tau = \frac{w_0}{\nu_s}$. For the acousto-optic deflector, its number of resolvable spots is proportional to the transition time ($N = \Delta f \cdot \tau$). When choosing the material for the deflector, the sound velocity $\nu_s$ is required to be low. So another quality factor $M_3$ is introduced:

$$M_3 = \left( \frac{1}{\nu_s} \right) M_1 = \frac{1}{\nu_s} (n^7 \nu_s^2)M_2 = \frac{n^7 P^2}{\rho \nu_s^2}$$  \hspace{1cm} (1.3-70)

Table 1.3-1 lists a number of acousto-optic materials and their physical properties.

### 1.3.4 The acousto-optic waveguide modulator

The structure schematic diagram of the acousto-optic Bragg diffraction-type waveguide modulator is as shown in Fig. 1.3-15. It is composed of a plane waveguide and a transducer with crossed electrodes. To effectively stimulate the surface elastic wave in the waveguide, piezoelectric materials (such as ZnO, etc.) are adopted for the waveguide and the underlay can be either piezoelectric material or non-piezoelectric material. For example, the underlay shown in Fig. 1.3-15 is $y$ cut up LiNbO₃ piezoelectric crystal material, the diffusive Ti waveguide, and the electro-acoustic transducer with crossed electrodes on the surface made by the photoetching method. The whole device can rotate round the $y$-axis to enable the included angle between the guided-wave light and the electrode strip to be regulated to a Bragg angle. The incident light now passes the waveguide via the input prism, while the
### Tab. 1.3-1 The properties of acousto-optic materials

<table>
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<tr>
<th>Material</th>
<th>λ/μm</th>
<th>n</th>
<th>ρ/(g/cm³)</th>
<th>( v_s/(10^3 \text{m/s}) )</th>
<th>( \mu ) direction</th>
<th>Light wave polarized direction</th>
<th>Quality factor</th>
<th>( M_1 = n^2 \rho^2 / \rho v_s )</th>
<th>( \star M_2 )</th>
<th>( \star \star M_3 )</th>
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<td>1.51 ( \times 10^{-15} )</td>
<td>1.29 ( \times 10^{-11} )</td>
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<td>( \perp ) or ( \parallel )</td>
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<td></td>
<td></td>
<td></td>
<td>arbitrary, polarized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bi₁₂SiO₂₀</td>
<td>0.63</td>
<td>2.30</td>
<td></td>
<td>Longitudinal, [100]</td>
<td>3.83</td>
<td>arbitrary</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(BSO)</td>
<td></td>
<td></td>
<td></td>
<td>arbitrary, polarized</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Sr₀.75Ba₀.25O₃</td>
<td>0.63</td>
<td>2.31</td>
<td></td>
<td>Longitudinal, [010]</td>
<td>2.31</td>
<td>2.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(SrN)</td>
<td></td>
<td></td>
<td></td>
<td>Longitudinal, [100]</td>
<td>2.31</td>
<td>2.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ge₃₃As₁₂Se₅₅</td>
<td>1.06</td>
<td>2.55</td>
<td>4.00</td>
<td>Longitudinal</td>
<td>2.50</td>
<td>246.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transparent lead glass</td>
<td>0.63</td>
<td>1.72</td>
<td>4.80</td>
<td>Longitudinal</td>
<td>3.80</td>
<td>10.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yellow lead glass</td>
<td>0.63</td>
<td>1.96</td>
<td>6.30</td>
<td>Longitudinal</td>
<td>3.10</td>
<td>16.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-methacrylate</td>
<td>0.63</td>
<td>1.55</td>
<td>1.18</td>
<td>Longitudinal</td>
<td>2.68</td>
<td>50.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.63</td>
<td>1.59</td>
<td>1.06</td>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NeKRS-5</td>
<td>0.63</td>
<td>2.60</td>
<td>7.37</td>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PbNO₃</td>
<td>0.63</td>
<td>1.78</td>
<td>4.70</td>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dimethyl benzene</td>
<td>0.63</td>
<td>1.50</td>
<td>0.86</td>
<td>Longitudinal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ M₁ = \frac{n^2 \rho^2}{\rho v_s}, \hspace{1em} M₂ = \frac{n^2 \rho^2}{\rho v_s} ; \hspace{1em} M₃ = \frac{n^2 \rho^2}{\rho v_s} \]
transducer-generated ultrasonic wave will cause periodic variation of the refractive indices in the waveguide and underlay. Therefore, relative to the acoustic wave front, after the light wave injected into the waveguide at an angle $\theta_B$ penetrates the output prism, level-1 diffracted light along the direction at an angle $2\theta_B$ with the primary light beam is obtained, whose intensity is

$$I_1 = I_i \sin^2 \left( \frac{\Delta \varphi}{2} \right) = I_i \sin(BV)$$

![Diagram of acousto-optic modulator](image)

Fig. 1.3-15 The acousto-optic Bragg waveguide modulator

where $\Delta \varphi$ is the phase retardation experienced by the guided-wave light passing through the length $L$ under the action of the electric field, and $B$ is a proportionality constant dependent on such factors as the waveguide effective refractive index $n_{eff}$, etc. The above formula shows that the refracted light intensity $I_1$ varies with the variation of voltage $V$, realizing the modulation of the intensity of the guided-wave light. For instance, when $\lambda = 0.6328 \, \mu m$, $V = 9 \, V$, 100% intensity modulation is obtained. The capacitance $C = 20 \, pf$, from which the driving power needed by unit bandwidth is 27 mW/MHz. When the frequency band is sufficiently broad, as the spatial locations of the output diffracted light are differently distributed, it can be used for optical deflection or for optical switching.

The Bragg modulated bandwidth is approximately inversely proportional to the acoustic wave aperture $L$. As $L$ cannot be taken too small, which will reduce the diffraction efficiency, the bandwidth of an actual device is restrained by this factor. So there should be a compromise when considering this factor. In addition, to obtain definite diffraction efficiency while increasing the bandwidth of the modulator, a change in the structure of the transducer is often made in actual application. That is, the equal period interdigital transducer can be changed to variable period interdigital in form (i.e., the spacing gradually changing along the direction of acoustic wave propagation). Owing to the fact that the efficiency of the electro-acoustic transducer is highest when the interdigital spacing is equal to the half-wave length of the acoustic wave, the variable period transducer generates acoustic waves of different wavelengths at different positions of digital strips, thus broadening the bandwidth of the transducer.

### 1.3.5 Acousto-optic deflection

Another important use of the acousto-optic effect is using it to make a light beam deflect. The structure of the acousto-optic deflector is basically the same as that of the Bragg light modulator, the difference being in that the modulator is intended to change the intensity of the diffracted light while the deflector is intended to change the direction of the diffracted light by changing the acoustic wave frequency to make it deflect so that not only is the light...
beam made to continuously deflect, but also the separate light spots can be made to deflect by scanning.

1. The principle of acousto-optic deflection

It is known from an analysis of the previously mentioned theory of acousto-optic diffraction that the generation of the maximum of diffraction by a light beam injected into a medium at angle \( \theta_i \) should satisfy the Bragg condition

\[
\sin \theta_B = \frac{\lambda}{2n\lambda_s}, \quad \theta_i = \theta_d = \theta_B
\]

The Bragg angle is in general very small and can be written as

\[
\theta_B \approx \frac{\lambda}{2n\lambda_s} = \frac{\lambda}{2nv_s f_s}
\]

Hence the included angle (deflection angle) between the diffracted light and incident light is equal to twice the Bragg angle \( \theta_B \), i.e.,

\[
\theta = \theta_i + \theta_d = 2\theta_B = \frac{\lambda}{nv_s f_s}
\]

It can be seen from the above equation that, by changing the frequency \( f_s \) of the ultrasonic wave, we can change its deflection angle \( \theta \), thereby attaining the goal of controlling the direction of light beam propagation. That is, the variation of the deflection angle of the light beam due to the change in the ultrasonic frequency \( \Delta f_s \) is

\[
\Delta \theta = \frac{\lambda}{nv_s} \Delta f_s
\]

This can be explained by Fig. 1.3-16 and the acousto-optic wave vector relationship. Suppose the acousto-optic diffraction satisfies the Bragg condition when the acoustic wave frequency is \( f_s \). Then the acousto-optic wave vector plot is a closed isosceles triangle and the diffraction maximum is along the direction at angle \( \theta_d \) with the ultrasonic wave face. If the acoustic wave frequency changes to \( f_s + \Delta f_s \), according to the relation \( k_s = \frac{2\pi}{v_s} f_s \), the acoustic wave vector quantity will have a change of \( \Delta k_s = \frac{2\pi}{v_s} \Delta f_s \). Since the incident angle \( \theta_i \) remains unchanged, as does the magnitude of the wave vector of the diffracted light, the acousto-optic plot will no longer be closed. The light beam will be diffracted along the OB direction, the corresponding light beam deflection is \( \Delta \theta_d \), and both the angles \( \theta \) and \( \Delta \theta \) are very small.

Hence it can be approximately considered that

\[
\Delta \theta = \frac{\Delta k_s}{k_s} = \frac{\lambda}{nv_s} \Delta f_s
\]

So the deflection angle is proportional to the change of the sound frequency.
1.3 Acousto-optic modulation

2. The main performance parameters of the acousto-optic deflector

There are three main performance parameters of the deflector, namely, the number of resolvable spots (which determines the capacity of the deflector), the deflection time $\tau$ (whose reciprocal determines the velocity of the deflector), and the diffraction efficiency $\eta_S$ (which determines the efficiency of the deflector). The diffraction efficiency has already been discussed earlier, so we shall mainly examine the number of resolvable spots, the deflection velocity, and the operating bandwidth.

For an optical deflector, not only should we study the magnitude of its deflection angle $\Delta \theta$, but, we should mainly consider the number of its resolvable spots $N$, which is defined as the ratio of the deflection angle $\Delta \theta$ to the diffusion angle $\Delta \phi$ of the incident light beam itself; that is,

$$N = \frac{\Delta \theta}{\Delta \phi} \quad (\Delta \phi = R\lambda/w) \quad (1.3-74)$$

where $w$ is the width of the incident light beam and $R$ is a constant, whose value depends on the property of the incident light beam (homogeneous light beam or Gaussian light beam) and the resolvability criterion (the Rayleigh criterion or resolvability criterion). For instance, for deflectors used in displaying or recording, the Rayleigh criterion is adopted, whose $R = 1.0 \sim 1.3$, while deflectors for optical memories adopt the resolvability criterion, whose $R = 1.8 \sim 2.5$. The number of resolvable spots by scanning is

$$N = \frac{\Delta \theta}{\Delta \phi} = \frac{w}{\nu_s} \frac{\Delta f_s}{R} \quad (1.3-75)$$

where $\frac{w}{\nu_s}$ is the transition time of the ultrasonic wave, denoted by $\tau$, which is exactly the deflection time of the deflector. So Eq. (1.3-75) can be written as

$$\frac{1}{\tau} = \frac{1}{R} \Delta f_s \quad (1.3-76)$$

$N\frac{1}{\tau}$ is called the capacity-velocity product of the acousto-optic deflector, which characterizes the number of resolvable positions the light beam can point to within unit time. The above equation shows that it only depends on the operation bandwidth $\Delta f_s$ while unrelated to the properties of the medium. Therefore, once the light beam width and sound velocity are determined, parameter $\tau$ is determined. Only when the bandwidth is increased, will it be possible to enhance the resolving power of the deflector. For instance, if the diameter of the incident beam $w = 1$ cm and the sound velocity $\nu_s = 4 \times 10^5$ cm/s, then $\tau = 2.5 \mu$s. If it is required that $N = 200$, then $\Delta f_s$ is $100 \sim 200$ MHz.

The bandwidth of the acousto-optic deflector is restrained by two factors, namely, the transducer bandwidth and the Bragg bandwidth because, when the acoustic frequency varies, so will the corresponding Bragg angle, the amount of variation being

$$\Delta \theta_B = \frac{\lambda}{2n\nu_s} \Delta f_s \quad (1.3-77)$$

So it is required that the acoustic beam and light beam possess a matched angle of divergence. Generally, the collimated parallel light beams are adopted for the acousto-optic deflector and, as its angle of divergence is very small, it is required that the angle of divergence of the acoustic wave $\delta \phi \geq \Delta \theta_B$. Taking $\delta \phi = \frac{\lambda_s}{L}$ and considering Eq. (1.3-77), we have

$$\frac{\Delta f_s}{f_s} \leq \frac{2n\lambda_s^2}{\lambda L} \quad (1.3-78)$$
In reality, the selection of the operation bandwidth is determined by the given indices $N$ and $\tau$, as is the central frequency of the operation frequency band. As it is in general not easy to make the $Q$ value of normal Bragg devices very big, there always exist some influences of surplus high-level diffraction and various nonlinear factors as well as the components of driving source harmonics. In order to prevent the appearance of false spots in the operating frequency band, according to inference and calculation, it is required that the central frequency of the operating bandwidth

$$f_{s0} \geq \frac{3}{2} \Delta f_s = 1.5 \Delta f_s$$

or

$$\frac{\Delta f_s}{f_{s0}} \leq \frac{2}{3} = 0.667 \quad (1.3-79)$$

This equation is the basic relational expression for designing the bandwidth of the Bragg acousto-optic deflector.

In order to enable the Bragg acousto-optic diffraction deflector to have good bandwidth characteristics, that is, to generate Bragg diffraction within a rather wide range of frequency and reduce as much as possible deviation from the Bragg condition, it is necessary to provide ultrasonic wave along an appropriate direction in a rather wide range of angle while doing everything possible to make the ultrasonic wave surface do the corresponding inclined revolution with the variation of the frequency so as to make the main direction of propagation of the ultrasonic wave equally share the direction of the incident light and that of the diffracted light from beginning to end so that the ultrasonic direction will automatically track the Bragg angle (referred to as ultrasonic tracking). The method of realizing ultrasonic tracking is in general to adopt a so-called “array transducer”; that is, by dividing the transducer into several pieces, the ultrasonic wave that makes them enter the acousto-optic medium is the ultrasonic wave emitted by all the transducers synthesized by superposing one over another to form an inclined wave surface. The primary direction of the synthesized ultrasonic wave varies with the variation of the sound wave frequency. Such a structure can ensure the fulfillment of the Bragg condition in a rather wide range of frequency. The array transducer is divided into the step type and plane type in terms of form. The structure of the former is shown in Fig. 1.3-17(a), which consists in grinding the acousto-optic medium into a series of steps; the height difference between steps is $\lambda S/2$, the width of a step is $S$. The pieces of transducer are each pasted onto a step. The phase difference between two adjacent pieces is $\pi$, so there is also a phase difference of $\pi$ rad between the surfaces of ultrasonic wave generated by each transducer. This makes the sound wave equiphase surface propagating
in the medium do inclined revolution with it, the angle of revolution varying with frequency. Such a situation is equivalent to a change in the angle of the incident light beam to make it satisfy the Bragg condition. The latter is a plane-type structure as shown in Fig. 1.3-17(b). Its operation is basically the same as the former and is omitted.

1.3.6 Matters for consideration in designing the acousto-optic modulator

According to the process of operation of the acousto-optic modulator, first the electric oscillation is converted to ultrasonic vibration by the electro-acoustic transducer. Then, by means of the adhesive layer between the transducer and the acousto-optic medium, the vibration is transferred into the medium to form the ultrasonic wave. Therefore, it is necessary to consider how to effectively convert the electric power provided by the driving source into ultrasonic wave power in the acousto-optic medium. Through the interaction between sound and light, the ultrasonic wave will obtain diffracted light by inducing Bragg diffraction of the incident light beam. For this reason, we just have to consider how to enhance its diffracting efficiency and in what frequency range modulation can be performed distortion-free, that is, how to make the design so that the ultrasonic wave along an appropriate direction can be provided in a rather wide frequency range, enabling both the incident light direction and the included angle \( \theta_i \) between the ultrasonic wave surfaces to meet the Bragg condition in this frequency range. That is, the method of design determines the possibility of enhancing the Bragg bandwidth. It is based on this requirement of the acousto-optic modulator that a definite analysis is made in this section on such issues as the selection of material for the acousto-optic medium, the design of the electro-acoustic transducer, etc.

1. The selection of material for the acousto-optic medium

The performance of the medium material exerts a direct influence on the quality of the modulator. Hence the great importance of rational selection of the medium material. During design, mainly the following factors should be considered:

(1) The modulating efficiency of the modulator should be high while the acoustic power needed should be as low as possible. The modulating efficiency of the modulator is characterized by the ratio of the post-modulation light intensity (i.e., diffracted light intensity) to the incident light intensity, or \( \eta_s = \frac{I_1}{I_i} = \sin^2 \left( \frac{\nu L}{2} \right) \), where \( \nu \propto M_2 \); that is, the greater \( M_2 \) is, the greater \( \nu \) will be, hence the high modulating efficiency. When choosing the acousto-optic material, material with great \( M_2 \) value should be chosen while making an overall consideration of the physical and chemical performance of the material.

For instance, if water is used as the acousto-optic medium, its characteristic parameters will be \( n = 1.33 \), \( P = 0.31 \), \( \nu_s = 1.5 \times 10^3 \text{ m/s} \), \( \rho = 1000 \text{ kg/m}^3 \), and the wavelength \( \lambda \) of the incident light is taken as \( \lambda = 0.6328 \mu\text{m} \). Substituting these values into Eq. (1.3-26), we have

\[
(\eta_s)_{\text{water}} = \sin^2(1.4L\sqrt{I_s})
\]

(1.3-80)

For other materials and other wavelengths, for convenience in calculation, the medium’s \( M_2 \) can be represented as the quality factor relative to water, denoted by \( M_{2(\text{material})} = M_{2(\text{water})} M_{2(\text{water})} \). Then Eq. (1.3-26) is expressed in a form suitable for an arbitrary wavelength, i.e.,

\[
\eta_s = \sin^2 \left( \frac{0.6328}{\lambda} L \sqrt{0.31I_s} \right)
\]

(1.3-81)

Listed in Tab. 1.3-2 are the \( M_{2(\text{water})} \) values of some frequently seen acousto-optic materials.
Tab. 1.3-2 The $M_\omega$ values of acousto-optic materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$/(kg/m$^3$)</th>
<th>$v_s$/(km/s)</th>
<th>$n$</th>
<th>$P$</th>
<th>$M_\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.00</td>
<td>1.50</td>
<td>1.33</td>
<td>0.31</td>
<td>1.00</td>
</tr>
<tr>
<td>Dense flint glass</td>
<td>6.30</td>
<td>3.10</td>
<td>1.92</td>
<td>0.25</td>
<td>0.12</td>
</tr>
<tr>
<td>Vitreosil (SiO$_2$)</td>
<td>2.20</td>
<td>5.97</td>
<td>1.46</td>
<td>0.20</td>
<td>0.006</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.06</td>
<td>2.35</td>
<td>1.59</td>
<td>0.31</td>
<td>0.80</td>
</tr>
<tr>
<td>KRS-5</td>
<td>7.40</td>
<td>2.11</td>
<td>2.60</td>
<td>0.21</td>
<td>1.60</td>
</tr>
<tr>
<td>Lithium niobate (LiNbO$_3$)</td>
<td>4.70</td>
<td>7.40</td>
<td>2.25</td>
<td>0.15</td>
<td>0.012</td>
</tr>
<tr>
<td>Lithium fluoride (LiF)</td>
<td>2.60</td>
<td>6.00</td>
<td>1.39</td>
<td>0.13</td>
<td>0.001</td>
</tr>
<tr>
<td>Titanium dioxide (TiO$_2$)</td>
<td>4.26</td>
<td>10.30</td>
<td>2.60</td>
<td>0.05</td>
<td>0.001</td>
</tr>
<tr>
<td>Sapphire (Al$_2$O$_3$)</td>
<td>4.00</td>
<td>11.00</td>
<td>1.76</td>
<td>0.17</td>
<td>0.001</td>
</tr>
<tr>
<td>Lead molybdate (PbMoO$_4$)</td>
<td>6.95</td>
<td>3.75</td>
<td>2.30</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>$\alpha$ iodide acid (HIO$_3$)</td>
<td>4.63</td>
<td>2.44</td>
<td>1.90</td>
<td>0.41</td>
<td>0.50</td>
</tr>
<tr>
<td>Tellurium dioxide (TeO$_2$)</td>
<td>5.99</td>
<td>0.617</td>
<td>2.35</td>
<td>0.09</td>
<td>5.00</td>
</tr>
</tbody>
</table>

If lead molybdate (PbMoO$_4$) is taken as an example, we find from the table that $M_\omega = 0.22$. Suppose the acoustic power $P_s = 1\ W$ ($I_s = 1\ W/mm^2$), the cross-sectional area of the acoustic beam is $1\ mm \times 1\ mm$, and the acousto-optic action length $L = 1\ mm$. Substituting the above-mentioned data into Eq. (1.3-81), we obtain

$$\eta_s = 40\%$$

(2) It is advisable to make the modulator possess a rather broad bandwidth. We already know that the Bragg condition is $\sin \theta_B = \frac{\lambda}{2v_s}$. Obviously, when the optical and acoustic wavelength vary, variation of the Bragg angle will be induced. In fact, the optical wave possesses definite frequency spectral width. When the modulator operates within a wide range of frequency band, the deviation of the acoustic frequency relative to the central frequency will cause the diffraction angle to deviate from the Bragg angle. When a definite value is exceeded, the operation state of the modulator will be made to not satisfy the Bragg condition, so that the level 1 diffracted light intensity becomes lower. The frequency variation $\Delta f_s$ to which half of the diffracted light intensity that the level-1 diffracted light intensity is lowered to relative to when it is at the central frequency is defined as the Bragg bandwidth. According to inference and proof, there is approximately $\Delta f_s = 1.8n v_s^2 \cos \theta_B \lambda L f_s$. So, the greater the $n v_s^2$ (i.e., $M_1 = n v_s^2 M_2$) is, the broader the Bragg bandwidth is. In order that the modulator will have a rather large bandwidth, a material with large quality factor $M_1$ should be chosen.

Of course, to obtain broadband modulation, apart from the requirements on the acousto-optic material, it is advisable to adopt the lens-focused fine Gaussian light beam when designing the acousto-optic modulator to make the transition time of the acoustic wave as short as possible. However, when the light beam emission angle $\Delta \theta_i$ is greater than the ultrasonic angle of divergence $\Delta \phi$, the rays at the edge will not be diffracted because of the absence of the light wave to satisfy the Bragg condition, affecting the performance of the modulator.

When assessing the performance of an acousto-optic medium, it is often necessary to consider the two indices, the bandwidth and the diffraction efficiency, at the same time. Hence the introduction of the parameter of efficiency bandwidth product ($\eta_s \Delta f_s$), i.e.,

$$\eta_s \Delta f_s \approx \frac{9 \pi n v_s^2 M_2}{\lambda^3 f_s H P_s} = \frac{9 M_1}{\lambda^3 f_s H P_s}$$

(1.3-82)

So when choosing the material, an overall consideration should be made of various factors
1.3 Acousto-optic modulation

prior to deciding on an appropriate material according to the specific requirements of the acousto-optic device.

2. The electro-acoustic transducer

The role of an electro-acoustic transducer is to transform the electric power into acoustic power so as to set up the ultrasonic field in a medium. Usually the anti-piezoelectric effect of a certain material is made use of to generate mechanical vibration under the action of an externally applied electric field. So it is not only a mechanically vibrating system but also an electrically oscillating system associated with an externally applied modulating source. Here we shall mainly analyze such issues as the mechanism of the electro-acoustic transducer, the electric characteristics, and the matching of mechanical vibration with an electric tank.

(1) The vibration equation of the transducer crystal. The $x$-0° cut-up quartz crystal wafer is in general adopted for the transducer, with an alternating electric field applied onto the quartz plate. When the direction of the electric field is the same as that of the piezoelectric axis, the crystal plate will stimulate elastically mechanical vibration along the direction of the thickness, as shown in Fig. 1.3-18. When the frequency of the electric field is equal to that of the inherent mechanical vibration of the crystal, the amplitude of the elastic vibration will reach maximum.

$$f_d = \frac{1}{2d} \sqrt{\frac{C_{11}}{\rho}}$$  \hspace{1cm} (1.3-83)

where $d$ is the thickness of the crystal wafer (cm), $\rho$ the density (g/cm$^3$), and $C_{11}$ the elastic modulus (N/m$^2$) of the vibrating form and orientation. For the quartz crystal, $\rho = 2.65$ g/cm$^3$, $C_{11} = 86.05 \times 10^4$ N/m$^2$. Thus, $f_d = \frac{285}{d}$ kHz ($d$ calculated in cm). It has been proved by experiment that, for the thickness-wise vibration of the quartz crystal wafer, it is suitable to adopt Eq.(1.3-84).

$$f_d = \frac{2580}{d}$$ kHz  \hspace{1cm} (1.3-84)

where $d$ is in mm. Thus, in actual application, it will be possible to calculate and determine the thickness of the crystal wafer according to the frequency needed. For instance, if it is required that the resonator frequency of the quartz transducer be 30 MHz, then the thickness of the crystal wafer $d = \frac{2880/(30 \times 10^3)}{\text{mm}} = 0.096$ mm.

The highest frequency that can be obtained by the piezoelectric quartz is about 50 MHz. Operating at such a high frequency and orientated to be normal to the $x$-axis, the quartz wafer has a thickness of merely 0.05 mm. So it’s very difficult in terms of manufacturing technology. Furthermore, when strongly stimulated, the crystal wafer may break from electric breakdown. So, if the transducer is required to obtain a higher frequency, this can be realized by having it operate in a higher harmonic state, actually proving that the quartz crystal wafer operating at an odd resonant frequency can satisfy the condition for resonant oscillation. Although the oscillation output of the transducer is decreased, such a method of stimulation makes it possible to increase the input electric power without running the risk
of electric breakdown. In particular, in certain applications where the power is not required to be very high, this method is especially suitable. The frequency of the $k$th-order harmonic vibration of the quartz crystal wafer is

$$f_k = kf$$

where $f$ is the natural frequency of the basic vibration. Table 1.3-3 shows the quartz wafer with orientations as shown in Fig. 1.3-18 ($L = H = 19.96$ mm, $d = 9.99$ mm), the higher-order resonant frequency $f_k$ measured, as well as the $f_k$ values calculated by formulae. The difference between the two $\Delta f = (f_k - f_k')$ is very small.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$f_k$/kHz</th>
<th>$f_k'$/kHz</th>
<th>$\Delta f$/kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>3170</td>
<td>3170</td>
<td>0.00</td>
</tr>
<tr>
<td>21</td>
<td>6061</td>
<td>6052</td>
<td>-0.43</td>
</tr>
<tr>
<td>31</td>
<td>8926</td>
<td>8934</td>
<td>-0.90</td>
</tr>
<tr>
<td>41</td>
<td>11816</td>
<td>11816</td>
<td>0.00</td>
</tr>
<tr>
<td>51</td>
<td>14713</td>
<td>14697</td>
<td>-0.90</td>
</tr>
<tr>
<td>61</td>
<td>20422</td>
<td>17579</td>
<td>-2.03</td>
</tr>
</tbody>
</table>

(2) The electric characteristics of the transducer crystal wafer. Although the vibration made by the electro-acoustic transducer is a mechanical one, it is driven by the electric oscillating energy as a load on the source. As it is equivalent to a subcircuit in the oscillating circuit, it is necessary to analyze its equivalent circuit and characteristics. The electrodes on the piezoelectric crystal wafer’s two surfaces along the direction of the $x$-axis (Fig. 1.3-18) are equivalent to a capacitor under the action of the source, which, represented by $C_1$, is

$$C_1 = \frac{\varepsilon_{11}A}{d}$$

where $\varepsilon_{11}$ is the dielectric tensor component of the piezoelectric crystal wafer, $A$ its area, and $d$ its thickness. When this capacitor is charged to voltage $V$, its electrical charge is equal to $q_0 = C_1V$. In addition, under the action of the electric field, the piezoelectric crystal will deform, which will lead to positive piezoelectric effect and so cause the surface of crystal wafer to generate electrical charge $q_1$. So the total charge on the two plates of the crystal is

$$q = C_1V + q_1$$

(1.3-85)

where $q_1 = d_{11}P$, where $d_{11}$ is the piezoelectric modulus and $P$ is the force of the electric field acting on the quartz wafer. When the driving voltage is $V = V_0 \sin \omega_m t$, the current flowing through the crystal wafer is

$$i = C_1 \frac{dV}{dt} + \frac{dq_1}{dt} = i\omega_m C_1 V_0 \cos \omega_m t + i_1$$

(1.3-86)

This equation shows that the piezoelectric crystal transducer in a circuit can be replaced by a capacitor $C_1$ and a parallel equivalent circuit, as shown in Fig. 1.3-19. According to inference and proof, the equivalent electric parameter of the crystal is obtained as follows:

$$L = \frac{\rho d^3}{8HLc_{11}^2}, \quad R = \frac{\pi^2 \rho pd}{8HLc_{11}^2}, \quad C = \frac{8\varepsilon_{11}^2 HL}{\pi^2 C_{11}d}, \quad C_1 = \frac{\varepsilon_{11}HL}{4\pi d}$$
1.3 Acousto-optic modulation

where $e_{11}$ is the piezoelectric constant, $\varepsilon$ the dielectric constant of the crystal, $\eta$ the mechanical loss constant of the medium, and $C_{11}$ the elastic modulus in the $x$-axis direction.

When a voltage is applied on the crystal, definite electric energy will be stored in it. Because of the piezoelectric performance of the crystal, part of the electric energy is consumed in the crystal to generate elastic stress and is transformed into mechanical energy. The ratio between the two kinds of energy is exactly the measure of the efficiency of the transducer. This ratio is called the electromechanical coupling coefficient. Under the condition of vibration along the direction of the thickness, the mechanical energy of every unit volume is equal to $\frac{1}{2}C_{11}d_{11}E_x^2$ while the electric energy of every unit volume is $\frac{\varepsilon E_x^2}{8\pi}$. Therefore,

$$k^2 = \frac{4\pi C_{11}d_{11}^2}{\varepsilon} \quad \text{or} \quad k = d_{11}\sqrt{\frac{4\pi C_{11}}{\varepsilon}} \quad \text{(1.3-87)}$$

For the quartz crystal, $k = 10\%$. The electromechanical coupling coefficient is an important parameter for characterizing the characteristics of the transducer, the value of which differs from crystal to crystal. So in application one always hopes to adopt a crystal with big $k$ values.

In order to be able to transfer the ultrasonic energy to the acousto-optic medium with no or fairly little loss, the sound impedance of the transducer should be as close to that of the medium as possible. This will reduce the reflection loss of the contact interface between the two. In fact, it is the rule for a modulator to introduce a transitional layer of coupled medium (metal or nonmetal) between the two. It can do three things, namely, it can transfer the ultrasonic energy into the medium, then it can reliably paste the transducer onto the medium, and, finally, it can play the role of the electrode of the transducer (if nonmetal is used as the coupling medium, an additional electrode must be used). If the requirement is that the sound impedance of the coupling medium match well the acousto-optic medium and the transducer, epoxy resin is in general adopted as the coupling adhering medium when the operation frequency is rather low. On the contrary, when the operation frequency is rather high, as it is possible that the thickness of epoxy resin may be close to the length of the sound wave, which will affect the response of the acousto-optic medium to the transducer-emitted ultrasonic wave, it is advisable to adopt a metallic material. Very often indium or an indium-tin alloy is adopted for experiments to obtain fairly good coupling effects. If indium and lead are alternately plated, not only can we enhance the emitting strength of the ultrasonic wave, the stability of the modulator can be improved as well.

3. Matching between the acoustic beam and light beam

As the incident light beam possesses a definite width, and the acoustic wave propagates in a medium with a limited velocity, the acoustic wave needs a definite transition time to penetrate the light beam. It is impossible for the response of the variation of the light beam intensity to that of the acoustic wave intensity to be instantaneous. In order to shorten its transition time to enhance its velocity of response, the modulator has the lens focus the light beam into the center of the acousto-optic medium when operating. The light beam becomes an extremely fine Gaussian light beam, thus reducing its transition time. As it is, to make full use of acoustic energy and light energy, it is considered that the relatively rational case of the
acousto-optic modulator is one in which it operates at the ratio of the angle of divergence of the acoustic beam to that of light beam \( \alpha \approx 1 \left( \alpha = \frac{\Delta \theta_i}{\Delta \phi} \right) \).

This is because, when the angle of divergence of the acoustic beam is greater than that of the light beam, the ultrasonic energy on its edge will be wasted. Conversely, if the light divergence angle is greater than the acoustic divergence angle, then the rays on the edge cannot be diffracted since there is no longer any ultrasonic of appropriate direction (i.e., one that satisfies the Bragg condition). So, in designing the acousto-optic modulator, the ratio of one to the other should be accurately determined. In general, the angle of divergence of the light beam is taken as \( \Delta \theta = \frac{4\lambda}{\pi d_0} \), where \( d_0 \) represents the diameter at the waist of the Gaussian light beam focused in the acousto-optic medium. The angle of divergence of the ultrasonic wave beam is \( \Delta \phi = \frac{\lambda s}{L} \), where \( L \) is the length of the transducer. Thus the ratio is found to be

\[
\alpha = \frac{\Delta \theta_i}{\Delta \phi} = \frac{4}{\pi} \left( \frac{\lambda L}{d_0} \right) \left( \frac{\lambda s}{\Delta \phi} \right) \tag{1.3-88}
\]

It has been proved by experiment that the performance of the modulator is best when \( \alpha = 1.5 \).

In addition, for the acousto-optic modulator, in order to enhance the extinction ratio of the diffracted light, in the hope that the diffracted light will be separated from the level-0 light as much as possible, it is also necessary to adopt the rigorous separability condition, that is, requiring that the included angle between the center of the diffracted light and the level-0 light be greater than \( 2\Delta \phi \), that is, greater than \( 8\lambda d \). As the included angle (i.e., the deflection angle) between the diffracted light and level-0 light is equal to \( \frac{\lambda s}{v_s} f_s \), the separability condition is

\[
f_s \geq \frac{8v_s}{\pi d_0} = \frac{8}{\pi} \frac{2.55}{\tau} \tag{1.3-89}
\]

or, because \( f_s = \frac{v_s}{\lambda s} \), it can also be written as

\[
\frac{1}{d_0} \leq \frac{\pi}{8\lambda s} \tag{1.3-90}
\]

Substitution of Eq. (1.3-90) into Eq. (1.3-88) yields

\[
\alpha = \frac{\lambda L}{2\lambda^2 s} \approx \frac{L}{2L_0} \tag{1.3-91}
\]

When the optimal performance condition for the modulator \( \alpha = 1.5 \), then

\[
L = 3L_0 \tag{1.3-91}
\]

from which the length \( L_0 \) of the transducer is determined. Then, by using Eq. (1.3-89), the diameter of the waist of the laser beam focused in the medium can be found:

\[
d_0 = \frac{v_s \tau}{f_s} = \frac{2.55v_s}{f_s} \tag{1.3-92}
\]

from which the focal length of the lens can be chosen.

### 1.4 magneto-optic modulation

#### 1.4.1 The magneto-optic effect

The magneto-optic effect is the physical basis of magneto-optic modulation. For some materials, such as the paramagnetic material, ferromagnetic material, and ferrimagnetic
material, the atoms or ions in their internal composition all have definite magnetic moments. The compounds composed of such magnetic atoms or ions and possessing powerful magnetism are called magnetic materials. It has been found that there are many small regions in the magnetic material, in each of which the magnetic moments of all the atoms or ions are arranged parallel to one another. These small regions are referred to as magnetic domains. As the magnetic moments of the magnetic domains are different in direction, the action of one would offset that of the other. So, macroscopically, no magnetism is exhibited. If along a certain direction of the object an external magnetic field is applied, then the magnetic moments of all the magnetic domains of the object will turn from different directions to the direction of the magnetic field. Thus magnetism is manifested externally. When the light wave passes through such a magnetized material, changes in its propagation characteristics will take place. This phenomenon is called the magneto-optic effect.

The magneto-optic effect includes the Faraday rotation effect, Kerr effect, the Cotton Mouton effect, etc., the most important of which being the Faraday rotation effect, which makes the polarization direction of a beam of linearly polarized light propagating in a medium under the action of an externally applied magnetic field rotate and the size of its polarization angle \( \theta \) proportional to the product of the magnetic field intensity \( H \) along the light beam direction and the length \( L \) of light propagating in the medium, i.e.,

\[
\theta = VHL
\]

where \( V \) is called the Verdet constant, which represents the angle the polarization direction has rotated after the linearly polarized light passes through the magneto-optic medium of unit length under unit magnetic field intensity. Table 1.4-1 lists the Verdet constants of some magneto-optic materials.

<table>
<thead>
<tr>
<th>Name of material</th>
<th>Coronal glass</th>
<th>Flint glass</th>
<th>Sodium chloride</th>
<th>Diamond</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>0.015 \sim 0.025</td>
<td>0.03 \sim 0.05</td>
<td>0.036</td>
<td>0.012</td>
<td>0.013</td>
</tr>
</tbody>
</table>

With respect to the physical reasons for the phenomenon of optical rotation, it can be explained that the externally magnetic field makes the moment of the medium molecules arranged in a fixed direction. When passing through it, a beam of linearly polarized light is resolved into two streams of circularly polarized light of identical frequency and identical initial phase, of which one rotates clockwise and is called the right rotating circularly polarized light whereas the other rotates counterclockwise and is referred to as the left rotating circularly polarized light. The two streams of circularly polarized light propagate with different velocities without acting on each other. Their phase retardations generated after they have passed through a medium of thickness \( L \) are, respectively,

\[
\varphi_1 = \frac{2\pi}{\lambda} n_R L, \quad \varphi_2 = \frac{2\pi}{\lambda} n_L L
\]

So, there exists a phase difference between the two streams of circularly polarized light.

\[
\Delta \varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda} (n_R - n_L) L
\]

After passing through the medium, they are synthesized as a stream of linearly polarized light, whose polarization direction has rotated an angle relative to that of the incident light.
In Fig. 1.4-1, YZ represents the vibration direction of the linearly polarized light injected into the medium, resolving the amplitude $A$ into the left rotating and right rotating vectors $A_L$ and $A_R$. Suppose the length $L$ of the medium has made the right rotating vector $A_R$ just rotate back to its original position. At this moment the left rotating light vector ($\nu_L \neq \nu_R$) has rotated to $A'_L$. So the synthesized linearly polarized light $A'$ has rotated an angle $\theta$ relative to the polarization direction of the incident light. This value is equal to half the angle $\delta$, i.e.,

$$\frac{\delta}{2} = \frac{\pi}{\lambda} (n_R - n_L) L$$

(1.4-3)

It can be seen that the polarizing direction of $A'$ will rotate rightward with the propagation of the light wave which is known as the rightward rotating light effect.

The rotating direction of the magnetically induced rotating light effect is only related to the direction of the magnetic field while independent of the direction of the rays propagating being forward or reverse, which is where the phenomenon of magnetically induced optical rotation is different from that of the natural optical rotation. During its round trip to and from a natural optically rotating material, the beam of light offsets itself since its rotating angles are equal but opposite in direction. But when passing through a magneto-optic medium, as long as the direction of the magnetic field remains unchanged, the rotating angle will always increase toward one direction. This phenomenon shows that the magnetically induced rotating light is an irreversible optical process. Hence its use in making such devices as the optical isolators and single-pass optical brakes.

Currently, the most frequently used magneto-optic material is mainly the yttrium iron garnet (YIG) crystal, whose absorption coefficient in the wavelength range $1.2 \sim 4.5 \mu m$ is very low ($\alpha \leq 0.03 \text{ cm}^{-1}$) and has a rather big Faraday rotation angle. This wavelength range includes the optimum range ($1.1 \sim 1.5 \mu m$) in fiber-optic transmission and the frequency range of certain solid lasers. Therefore, it is possible to make magneto-optic devices such as modulators, isolators, ring-shaped devices, etc. using this material. As the variation of its physical performance with the temperature is not great, the magneto-optic crystal is not easily air-slaked and the modulating voltage is low. This is where it is superior to the electro-optic and acousto-optic devices, but when the operation wavelength has exceeded the above-mentioned range, the absorption coefficient will abruptly increase, even to the extent of making the device inoperable. This shows that it is in general not transparent in the region of visible light, while only usable in the near-infrared region and infrared region, which greatly limits its application.

1.4.2 The magneto-optic volume modulator

Similar to the electro-optic and acousto-optic modulation, the magneto-optic modulation also consists in having the information to be transmitted transformed into the variation of the intensity (amplitude) of the light carrier wave, the difference being that the magneto-optic modulation converts the electric signal into an alternating magnetic field corresponding to it first, with the polarized state of the light wave transmitted in the medium changed by the magneto-optic effect so as to attain the goal of changing the parameters like the light intensity, etc. The composition of the magneto-optic volume modulator is as shown in
1.4 magneto-optic modulation

Fig. 1.4-2. The operation material (YIG or Ga-doped YIG rod) is placed on the optical path along the axial direction, at whose two ends are placed the polarizing and analyzing devices, with the high frequency spiral-shaped coil wound around the YIG rod and controlled by the driving source. In order to obtain linear modulation, apply a constant magnetic field $H_{dc}$ on the direction normal to light propagation, its intensity sufficient to make the crystal saturation magnetized. During operation, the high frequency signal current will induce and generate a magnetic field parallel to the direction of light propagation when passing through the coil. When the incident light passes through the crystal, because of the Faraday rotation effect, its polarized plane is seen to rotate, its rotation angle directly proportional to the magnetic field intensity $H$. Therefore, so long as the modulating signal is used to control the variation of the magnetic field intensity, the polarized plane of light will be made to undergo corresponding variation. But with the constant magnetic field $H_{dc}$ here applied, which is normal to the light transmitting direction at that, the rotation angle is inversely proportional to $H_{dc}$. Thus

$$\theta = \theta_s \frac{H_0 \sin(\omega t)}{H_{dc}} L_0 \quad (1.4-4)$$

where $\theta_s$ is the unit length saturated Faraday rotation angle and $H_0 \sin \omega t$ the modulating magnetic field. If the analyzer is also used, we can obtain the modulated light with definite intensity variation.

1.4.3 The magneto-optic waveguide modulator

Figure 1.4-3 shows the structure of a magneto-optic waveguide mode transforming modulator. On the disc-shaped yttrium-gallium garnet (Gd$_3$Ga$_5$O$_{12}$-GGG) substrate, the epitaxial growth Ga- and Se-doped YIG magnetic thin film as the waveguide layer (thickness $d = 3.5 \, \mu m$, $n = 2.12$). On the surface of the magnetic thin film a metallic snake-shaped circuit is made using the phototetching technique. When an electric current flows through the snake-shaped circuit, the electric current in a certain channel of the snake-shaped circuit is along the $y$ direction, the current in the adjacent channel is along the $-y$ direction, and this current is capable of generating a magnetic field with the $+z$ and $-z$ directions alternately changing. Then there may appear in the magnetic thin film the alternate saturation and magnetization along the $+z$ and $-z$ direction. Suppose the period of the magnetic field variation (i.e., the period of the snake-shaped structure) is

$$T = \frac{2\pi}{\Delta \beta}$$

where $\Delta \beta$ is the difference between the TE mode and TM mode propagation constants. Owing to the mismatch of the lattice constant with thermal expansion between the thin film and the substrate, the magnetization-prone direction is located within the plane of the thin
film, the demagnetizing factor in which is zero. So the magnetization intensity $M$ can be made to revolve freely in the thin film plane with a small magnetizing field. If laser ($\lambda = 1.152 \, \mu m$) is input and output by two prism couplers, what is injected in is the TM mode. Because of the Faraday magneto-optic rotation effect, with the transmission of the light wave along the $z$ direction (the direction of magnetization) in the waveguide thin film, the electric field vector (along the $x$ direction) originally located within the thin film will turn to the normal direction (the $y$ direction). That is, the TM mode is transformed into the TE mode. Since the magneto-optic effect is proportional to the component $M_z$ of the intensity of magnetization $M$ along the direction of propagation $z$, changing the electric current in the snake-shaped circuit by applying a direct current magnetic field $H_{dc}$ between the $z$-axis and $y$-axis along the 45° direction can change $M_z$, thus changing its transforming efficiency. When the input electric current is so large that it makes $M$ saturate along the $z$ direction, the transforming efficiency will reach maximum. If the device’s $T = 2.5 \, \mu m$, a direct current of 0.5 A is input in the snake-shaped circuit, and the magneto-optic interaction length $L = 6 \, \text{mm}$, then 52% of the power of the input TM mode ($\lambda = 1.152 \, \mu m$) transformed to the TE mode. The output coupler of the magneto-optic waveguide mode transforming modulator is a rutile prism with a high birefringence. No matter whether the output TE and TM modes are resolved into two light beams with a field angle of 20°11′ each, or when the frequency of the electric current input in the snake-shaped circuit is from 0 to 80 MHz, the modulation of the light intensity in the two modes can be observed just the same.

### 1.5 The direct modulation

By the direct modulation is meant transforming the information to be transmitted into electric current signal that is injected into the semiconductor light source (the laser diode LD or semiconductor diode LED) so as to obtain the modulated signal. As it is carried on in the interior of the light source, it is also called the internal modulation. Such a direct modulation is both convenient and highly efficient besides being capable of high-speed modulation. It is the practical modulating method universally used in the fiber-optic communication system. In terms of the type of the modulating signal, the direct modulation can also be divided into analog modulation and digital modulation, the former carrying out direct modulation of the light intensity for the light source using continuous analog signals (e.g., TV, voice signals, etc.) and the latter carrying out the intensity modulation for the light source using the pulse code modulated (PCM) digital signals. Below we shall make a brief description of the two modulating methods.
1.5 The direct modulation

1.5.1 The principle of direct modulation by the semiconductor laser (LD)

The semiconductor laser is a device by means of which electrons and photons act on each other and perform direct energy transformation. Figure 1.5-1 shows the curve of relationship between the output light power and the driving current of the arsenic-gallium-aluminum (AlGaAs) double heterojunction injection laser. The semiconductor laser has a threshold electric current $I_t$; when the driving current density is smaller than $I_t$, the laser basically does not shine or merely emits very faint fluorescent light of very broad spectral line width and rather poor directionality. When the density of the driving current is greater than $I_t$, however, it begins to emit laser. Now the width of the spectral line and the direction of radiation become evidently narrower while the intensity greatly increases. Furthermore, with an increase in the electric current, it appears to increase linearly, as shown in Fig. 1.5-2. It can be seen from Fig. 1.5-1 that the strength of laser emission is directly related to the magnitude of the driving current. If the modulating signal is applied on the laser (power source), it will be possible to directly change (modulate) the intensity of the laser’s output light. By virtue of its simplicity and ability to operate at high frequency plus the guarantee of a good linear operation region and bandwidth, this modulating manner has found wide application in fiber optic communication, optical disk, and optical duplication, etc.

![Fig. 1.5-1 The output characteristics of semiconductor laser](image1)

![Fig. 1.5-2 The spectroscopic characteristics of semiconductor laser](image2)

What is shown in Fig. 1.5-3 is a schematic diagram of the modulating principle of the semiconductor laser, in which (a) shows the schematic of the electrical principle while (b) shows the curve of the relationship between the output optical power and the modulating signal. In order to obtain linear modulation so that the operating point will be located at rectilinear part of the output characteristic curve, it is necessary to apply a suitable biased current $I_b$ while introducing the modulating signal current so that the optical signal output will not become distorted. But care must be taken that the modulating signal source be isolated from the direct current bias source to prevent the latter from affecting the former. When the frequency is rather low, this can be done by connecting the capacitance to the inductance coil in series. When the frequency is very high (>50 MHz), the high pass filter circuit must be adopted. In addition, the biased electric current directly affects the modulating performance of the LD. Usually, $I_b$ should be chosen to be in the vicinity of the threshold current and slightly lower than $I_t$. Then the LD can obtain a fairly high modulating efficiency because, such being the case, the LD has no need for time to prepare for continuous emission of light signals (i.e., the delay time is very short), its modulating rate not limited by the average life of the carrier in the laser. At the same time, the relaxation oscillation, too, will be restrained to some extent. However, if the $I_b$ is chosen too high, the
extinction ratio of the laser will be deteriorated. So an all-around consideration should be made when choosing the bias current.

![Semiconductor Laser Modulating signal](image)

When the semiconductor laser is in the state of continuous modulating operation, the power loss is rather large owing to the direct current bias, thus inducing a rise in temperature which will affect or damage the normal operation of the device. Now the emergence of the double heterojunction laser has made the threshold current density greatly decrease compared with that of the homojunction laser, making it possible to satisfactorily operate in the mode of continuous modulation at room temperature.

To prevent modulation distortion when the semiconductor laser is performing modulation at a high frequency, the fundamental requirement is that the output power maintain a good linear relation with an electric current above the threshold. In addition, in order to prevent the emergence of relaxation oscillation as much as we can, it is advisable to adopt a laser structure of rather narrow strip width. Furthermore, direct modulation will reduce the strength of the laser’s principal mode while increasing that of the second mode, thus broadening the spectral line of the laser. Meanwhile, the pulse width $\Delta t$ generated by modulation and the spectral line width $\Delta \nu$ constrain each other, constituting the so-called bandwidth limit to the Fourier transform. Therefore, the capability of the semiconductor laser for direct modulation is restricted by the $\Delta t \cdot \Delta \nu$ product. So, for modulation at a high frequency, one had better adopt a modulator of the quantum well structure or an external modulator.

### 1.5.2 The modulating characteristics of the semiconductor light-emitting diode (LED)

As the semiconductor light-emitting diode is not a threshold device, unlike the semiconductor laser, its output light power will not undergo an abrupt change with the variation of the injected electric current. Hence the linearity of the LED’s $P-I$ characteristic curve is fairly good. Figure 1.5-4 shows a comparison of the $P-I$ characteristic curve between LED and LD. It can be seen from the figure that LED$_1$ and LED$_2$ are the $P-I$ characteristic curves of the front luminous-type light-emitting diodes while LED$_3$ and LED$_4$ are those of the end face luminous-type light-emitting diodes. It can be seen from the figure that the $P-I$ characteristic curve of the light-emitting diode is obviously better than that of the
1.5 The direct modulation

A semiconductor laser. So it has found wide application in the analog fiber optic communication system. But in the digital fiber optic communication system, because of its incapability of obtaining very high modulating rate (the highest being merely 100 Mb/s), its application is limited.

1.5.3 The analog modulation of the semiconductor light source

No matter whether the LD or the LED is used as the light source, it is required that a bias current be applied to make the operating spot located in the straight line section of the $P-I$ characteristic curve of the LD or LED, as shown in Fig. 1.5-3(b) and Fig. 1.5-5(b). The quality of the modulated linearity is related to the modulating depth $m$.

$$m = \frac{\text{modulating current amplitude}}{\text{bias current} - \text{threshold current}}$$

For LED:

$$m = \frac{\text{modulating current amplitude}}{\text{bias current}}$$

Fig. 1.5-4 The LED and LD $P_{out}$-$I$ curves compared

It can be seen from the figure that when $m$ is great, the amplitude of the modulating signal will be great. So, the linearity will be poor. Conversely, if $m$ is small, although the linearity is good, the amplitude of the modulating signal will be small. Therefore, a suitable $m$ value should be chosen. Moreover, in the analog modulation, the linear characteristic of the light source device itself is the principal factor in determining the quality of the analog modulation. So, in applications requiring rather high linearity, it is necessary to make nonlinearity compensation; that is, the nonlinear distortion due to the light source should be corrected by the electronic technology.

1.5.4 The digital modulation of the semiconductor light source PCM

Digital modulation is the modulation of the optic carrier wave emitted by the light source using the binary signal code “1” and code “0”, while digital signals very often adopt pulse code modulation (PCM), that is, first, the continuous analog signals are transformed into a group of amplitude modulated pulse sequences by “sampling”. Then, through the processes
of “quantization” and “coding”, they form a group of rectangular pulses of equal amplitude and equal width as “code elements”. For instance, the different combinations of “with pulse” and “without pulse” (with definite digit capacity of pulse code elements) represent the amplitude of the sampled value. This is pulse coding, which results in transforming the continuous analog signals into PCM digital signals, called “analog/digit” or A/D transformation (see books on relevant fiber optic communication for the specific process). Then, the PCM digital signal is used to perform intensity modulation for the light source, the characteristic curve of whose modulation is as shown in Fig. 1.5-6.

Because of its remarkable merits, digital optical communication has very good prospects for application. To begin with, the noise and distortion introduced in the course of transmission of the digital optical signals in the channel can be removed by adopting an indirect relay. Therefore it has strong anti-interference capability. Second, it does not make high requirements on the linearity of the digital fiber communication system, with the light-emitting power of the light source (LD) made full use of. Third, the digital optical communication equipment is conveniently connected to the PCM telephone terminal, the PCM digital color TV terminal, and the computer terminal, thereby making up a comprehensive communication system capable of transmitting not only telephone and color TV information but also computer data.

1.6 The spatial light modulator

1.6.1 The basic concept of the spatial light modulator

The different kinds of modulators described previously consist in exerting an action on a beam of light as a “whole” and, for each and every point on the \( x \)-\( y \) planes normal to the direction of light propagation, the effect is identical. The spatial light modulator (SLM) can form amplitude (or intensity) transmissivity varying with the \( xy \) coordinate.

\[
A(x, y) = A_0 T(x, y)
\]

Or it may form phase distribution varying with the coordinate.

\[
A(x, y) = A_0 \exp[i\theta(x, y)]
\]
1.6 The spatial light modulator

Or it may form different scattering states varying with the coordinate. Judging by its name, this is a kind of device that carries out modulation of the spatial distribution of the light wave. It contains many independent elements (called pixels) that are arranged as one- or two-dimensional arrays, each capable of independently accepting the control by the light signal or electric signal and changing its own optical properties (transmissivity, reflectivity, refractive index, etc.) in accordance with the signal, thus performing modulation of the light wave passing through it. The signal that controls the optical properties of these pixels is called the “write in signal” (W), which can be either the optical signal or the electric signal; the light wave injected into the device and modulated is called the “read out light” ($I_R$); the output light wave after going through the spatial light modulator is called the “output light” ($I_O$), as shown in Fig. 1.6-1.

![Schematic diagram of the spatial light modulator](image)

Obviously the write in signal should contain the information about the control over the pixels of the modulator and transmit each piece of the information to the modulator’s corresponding pixel position to change its optical properties. When the write in signal is a light signal, it is usually represented as a two-dimensional image of light intensity distribution, which is imaged on the pixel plane of the spatial light modulator via an optical system. This process is spoken of as addressing, which, in this way, is called optical addressing. Since in terms of time addressing by all pixels is accomplished at the same time, optical addressing is a parallel addressing mode. When the read out light passes through the modulator, its optical parameters (amplitude, intensity, phase, or polarized state) are modulated by the various pixels of the spatial light modulator. As a result, it becomes a beam of output light with new optical parametric spatial distribution. This mode is mainly used in light-light transform devices, which can be applied in optical information processing and in optical computers for image transformation, display, storage, or filtering. In particular, when performing real-time two-dimensional parallel processing to exhibit the advantages of optical information processing, there is even a greater need for the real-time spatial light modulator. When the write in signal is an electric signal, it is necessary to adopt the “electric addressing” mode because the electric signal is a time sequence. In principle, the signals can only be output to the pixels of the modulator one by one. So the electric addressing is a mode of serial addressing, which is mainly used for the electro-optical real-time interfacing devices. Its merit is its capability of directly using the electric signal to control the amplitude or phase of the output light, getting easily connected to the computer as well as to electronic analog signals such as the TV camera signals. Under the action of the write in signal, the optical properties (transmissivity, refractive index, reflectivity, optical activity, and surface deformation, etc.) of the pixels of the spatial light modulator will vary accordingly. The mechanisms that cause the variation of the optical properties are mainly the electro-optic effect, acousto-optic effect, and magneto-optic effect as well as the electric absorption effect of such materials as all kinds of crystals, liquid crystals, and organic polymers (the principles of which are as treated previously).

1.6.2 The basic functions of the spatial light modulator

The basic function of the spatial light modulator is to provide real-time or quasi-real-time one-dimensional or two-dimensional optical sensing devices and operating devices. Different types of spatial light modulators each have their own characteristics, but they have some common or similar performance and functions which, summed up, are as follows:
Chapter 1 Laser Modulation and Deflection Technology

1. The function of a transducer

In the electro-optic mixed processor, the write in electric signals can be transformed into output light signals, and such an output can be either one- or two-dimensional data groups arranged in the format needed or two-dimensional images. For instance, the to-be-processed information is from the analog signals of a camera or computer, which is often an electric signal varying with time. To input the signal into an optical processing system, we have to use a spatial light modulator. On the one hand, the serial electric signals arranged in the order of time are converted to control signals arranged in the form of one- or two-dimensional arrays and, on the other, the control signal on each pixel in the array is converted to one capable of modulating the read out light following the change in its optical properties.

In a real-time processing system, a write in non-coherent light signal can be converted to an output coherent light signal. As the object for processing by a real-time processing system is often an actual object, an ordinary optical system can only make it form a non-coherent image. But the processing system, on the contrary, requires a coherent image for frequency domain processing or light interference-based processing, etc. For example, in Fig. 1.6-2, the write in signal $I_W$ is a two-dimensional image made up of a beam of non-coherent light while the read out light $I_R$ is a beam of coherent light with a uniform amplitude. Then, when the spatial light modulator adopts the optical addressing mode to transform the write in light illuminance distribution into the light intensity transmission coefficients of the pixels, its output light $I_O$ is a beam of coherent light carrying the write in image information.

![Fig. 1.6-2 The principle of optical addressing spatial light modulation](image)

2. The amplification function

When the write in light intensity is rather weak, or when the image signal becomes weak in the process of information processing, it can be strengthened by adopting a spatial light modulator with spatially uniformly distributed read light of high intensity to obtain amplified output coherent light signals.

3. The operation function

For most spatial light modulators, signal multiplication is its intrinsic performance. As is shown in Fig. 1.6-1, the read light $I_R$ carries the information on a two-dimensional image, with the write in signal $I_W$ for controlling the transmittance of the spatial light modulator’s pixels. Thus, the light intensity distribution of the output light signal $I_O$ on the surface of the spatial light modulator is equal to the product of $I_O$ and the signal $I_W$. If the write in signal represents a matrix, and the read out light another, then multiplication between matrices of numbers can be realized using the spatial light modulator. In addition, operations related...
to the basic multiplying functions can also be performed, for example, programmable match filtering, the computer-controlled reconstructible optical interconnection, etc.

4. The threshold operation function

By means of the threshold characteristics of devices, a continuous write in signal can be converted to a number of separate “values” to output. The simplest operation is quantize the write in signal as 0, 1 to output, with a threshold given. When the write in signal exceeds the threshold, the output is “1”, but when the write in signal is smaller than the threshold, the output is “0” (i.e., there is no output light). Such an operation is spoken of as the threshold operation. By utilizing this characteristic, it is possible to realize the binary logic operation and A/D transformation. A spatial light modulator that exhibits the threshold characteristic can be regarded as a two-dimensional array of the nonlinear optical switching.

Apart from the above-mentioned functions, the spatial light modulator has such functions as short-term storage (memory), optical amplitude-restriction, wave face recovery, etc., which are not dealt with here.

1.6.3 Several typical spatial light modulators

1. The Pockels readout optical modulator

An optical addressing-type spatial light modulator made by using the electro-optic effect, the Pockels readout optical modulator (PROM) is currently in application for its good performance.

(1) The structure of the Pockels readout optical modulator

To meet the requirement on real-time processing, there have emerged devices of a multitude of structural principles one after another. Some are manufactured by combining the photosensitive thin film with ferroelectric crystal, some by utilizing the photoconductive crystals that possess photosensitive performance themselves, of which the spatial light modulators made of Bi$_{12}$SiO$_{20}$(BSO) for short) have been developing fairly quickly. BSO is a kind of noncentrosymmetric cubic crystal (of point group 23) that possesses not only the photoconductive effect, but also the linear electro-optic effect. It has a rather low half wave voltage and is fairly sensitive to the blue light of $\lambda = 400 \sim 450$ nm (as blue light has a very high photon energy) while its photoconductive effect on red light of 600 nm is very weak. As the characteristic of photosensitivity varies abruptly with the difference in wavelength, the material is sensitive to blue light but insensitive to red light. So blue light can be used as the write in light while red light can be used as the read out light, thereby reducing the interference with each other.

A schematic diagram of the structure of the BSO-PROM spatial light modulator is as shown in Fig. 1.6-3.

![Fig. 1.6-3 Schematic diagram of the structure of reflective BSO spatial light modulator:](image)

1.6-transparent electrodes; 2,5-insulating layers; 3-bichromatic reflective layer; 4-BSO crystal

The two sides of the BSO crystal are coated with a 3 $\mu$m thick insulating layer and the outermost layer is plated with transparent electrode to make it a transmissive device. If the write in side is plated with a two-color reflective layer to reflect the red light and let the blue light penetrate, then we have the reflective device. The reflective structure can not
only reduce the half-wave voltage, but also it has removed the influence of optical activity of the crystal itself.

(2) The operation principle of the BSO-PROM spatial light modulator

The principle consists in converting the light intensity distribution of the image into spatial distribution of the voltage applied on the BSO crystal, thereby transmitting the image onto the readout light beam, the former utilizing the crystal’s opto-electric conductivity and the latter, its Pockels electro-optic effect. The specific process of operation is: if there is no illumination after the operation voltage is applied on the transparent electrode, there is no variation of the optical properties of the crystal. Since the value of the photosensitive layer’s resistance is very great, the greater part of the voltage having fallen on the photosensitive layer. If powerful blue light is used to illuminate the photosensitive layer, then the photons are excited and the electrons are enabled to obtain sufficient energy to surmount the forbidden zone and jump into the conductive zone. So there will be large quantities of free electrons and cavities taking part in conduction to make the resistance in the photosensitive layer decrease to scantiness (called the photoconductive effect). Thus the greater part of the voltage is applied on the BSO crystal. Since the resistance value of the photosensitive layer varies with the intensity of the incident light from without, the electro-optic effect of the crystal, too, undergoes corresponding variation with the intensity of the incident light. For instance, use a beam of laser to carry image information as the write in signal $I_W$ for injection to the device from the right of the figure, to be illuminated onto the BSO crystal. As the electron-cavity pair is excited by the opto-electric effect in the crystal, the electrons are pulled toward the positive pole while the cavity causes the spatial variation of the potential according to the image shape distribution of the write in light. Thus, the illuminance distribution of the write in light is turned into electric field distribution in the BSO crystal via the opto-electric effect so that the image is stored. When fetching an image, use long wave light, such as the red light of 633 nm as readout light $I_R$ to illumine the device from the left of the figure via the polarizer (in the $x$ direction). The light becomes elliptically polarized light owing to the electro-optic effect, its ellipticity depending on the spatial variation of the voltage in the crystal. Therefore, the light intensity distribution from the analyzer (in the left of the figure, placed orthogonal to the polarizer) output light $I_O$ will be proportional to the bright-and-dark distribution; that is, the spatial modulation of light is realized.

The process of operation of the above-mentioned electro-optic spatial modulator is as shown in Fig. 1.6-4. What is shown in Fig. 1.6-4(a), (b), and (c) represents the preparatory stage before write in. Shown in Fig. 1.6-4(a) is the application of voltage $V_o$ between the two electrodes of the crystal; shown in Fig. 1.6-4(b) is illuminating the photosensitive layer with uniform lamp light to make it generate electron-cavity pairs and drift towards the electrode interface of the crystal under the action of the external electric field to make the electric field in the crystal zero, that is, eliminate the originally stored images (as the hidden resistance of the BSO is very great, the images stored can be kept for a very long period of time). What is shown in Fig. 1.6-4 is the reversal of the voltage to make the voltage on the crystal rise to $2V_o$ while Fig. 1.6-4(d) shows the write-in situation, where blue light of a rather short wavelength is used to carry the image information as the write in light $I_W$ to be imaged on the surface of a BSO crystal wafer, which is converted into electric field distribution within the crystal via the opto-electric effect and then into the distribution of double refractive index via the electro-optic effect. Figure 1.6-4(e) shows the read out situation in which the linearly polarized red light of long wavelength is used as the readout light $I_R$. The reason for choosing red light of a long wavelength as the read out light is because it is basically incapable of generating the opto-electric effect on the BSO crystal, thus unable to destroy the originally written in image of the electric field. Because of birefringence, it is resolved into two mutually normal polarized components after it is injected into the crystal. As there
is a phase difference between the two, the polarization state of their synthetic light varies accordingly. Therefore the light $I_O$ output from the analyzer is the intensity modulated light. In the bright region of the recording screen, which is the unexposed region of the crystal, the crystal’s birefringence effect is very weak. As the polarization state of the light beam in this region is almost unchanged, there is no display of image.

Fig. 1.6-4 The BSO-PROM spatial light modulator

2. The liquid crystal spatial light modulator

For an ordinary crystal, there is a definite melting point at which it changes from the solid state into the liquid state. Below the melting point it is in the solid state while above it, the liquid state, with the properties of a crystal lost at that. But some materials do not directly change from solid into liquid, but do so via a transitional phase. Now, on the one hand, it possesses the property of fluidity of liquids and, on the other, it has the characteristics of a crystal (e.g., optical, mechanical and thermal anisotropy). This transitional phase is spoken of as the “liquid crystal”.

The liquid crystal is an organic compound composed in general of rod-shaped cylindrical symmetric molecules. It possesses a very strong electric dipole moment and an easily polarized chemical group. Owing to the anisotropic characteristics of the liquid crystal’s molecules in terms of shape, dielectric constant, refractive index, and electric conductivity, when an external (electric, thermal, magnetic, etc.) field is applied on such a material, changes in the liquid crystal molecules’ direction of arrangement and flowing position will take place, or the physical state of the liquid crystal will be changed. For instance, if an electric field is applied on it, its optical properties will change. This is the liquid crystal’s electro-optic effect, which mainly includes the twisted effect, dynamic scattering effect, electro-controlled birefringence effect, phase change effect, guest-host effect, etc.

The fairly typical applied device of the liquid crystal modulator is the cadmium sulfide (CdS) optical valve. Its structural schematic is as shown in Fig. 1.6-5 along with and the
plate glass that is used for the purpose of maintaining the fixed shape of the device. The transparent electrode material is a mixed material indium-tin oxide (ITO) of indium oxide (In$_2$O$_3$) and tin oxide (Sn$_2$O$_3$), the material for the liquid crystal molecule orientation film layer is SiO$_2$, which makes the liquid crystal molecule thin film in contact with it arrange itself along the plane; the reflectivity of the multi-layer medium membrane reflective mirror is about 90%, which is also used as the insulator between the two transparent electrodes to prevent the direct current from flowing through the liquid crystal layer; the material of the optically isolated layer is CdTe, which keeps the write in light $I_W$ injected from the right side from injecting the left side of the optically isolated layer, while keeping the light leaking out from the reflective film from injecting the optical guide layer so as to isolate the write in light from the read out light. The material for the optical guide layer is CdS, the action of which will be described below.

![Fig. 1.6-5 The structure of the CdS liquid crystal optical valve; 1-medium film; 2,12-plate glass; 3,11-transparent electrodes; 4,7-liquid crystal molecular orientation film layer (SiO$_2$); 5-liquid crystal; 6-insulating ring; 8-multilayer medium reflective mirror; 9-light isolating layer; 10-photoconducting layer (CdS); 13-power source]

The main function of the liquid optical valve under discussion is the realization of the non-coherent/coherent transformation of images, the operation process of which consists in making the to-be-transformed non-coherent image imaged onto the photoconductive layer from the right side of the device via an optical system (as the write in light $I_W$), with a beam of linearly polarized coherent light (as the read out light) injected toward the liquid crystal layer from the left side of the device, its polarizing direction consistent with the direction of the molecular long axis at the left end of the liquid crystal layer. Because of the action of the highly reflective film, this beam of light will twice pass through the liquid crystal layer to finally go out from the left and, by means of an analyzer, the direction of whose polarizing axis is normal to the direction of the $I_R$ polarization to obtain the output light $I_O$.

When the power source applies the voltage on the series entity of the liquid crystal layer, the highly reflective film, the light isolating layer, and the photoconductive layer via the two transparent electrodes, as both the light isolating layer and the highly reflective film are very thin and the alternating current impedance is rather low, the voltage mainly falls on the liquid crystal layer and the photoconductive layer. Obviously, the proportion of voltage allocation on the two layers depends on the illumination of the photoconductive layer. For the dark portion in the incident light image, as the photoconductive layer is
not illuminated and the electric conductivity is very low (i.e., the resistance is very great), the voltage is mainly allocated on the photoconductive layer while the voltage obtained by the liquid crystal is scanty, which is insufficient to generate appreciable electro-optic effect. Therefore, in the corresponding position, the liquid crystal is still in the original state (i.e., with a structure of arrangement of a 45° twist). After the read out light passes through this position, its output light intensity $I_o$ is zero. For the pixel position of maximum illuminance in the incident light image, because of the internal opto-electric effect, the impedance of the photoconductive layer drastically diminishes, with the greater part of the voltage falling on the corresponding position of the liquid crystal layer, thus generating an apparent electro-optic effect. When the read out light passes through this position, its output light $I_o$ is maximum. Then, for the pixel position of other densities of illuminance in the incident light image, the corresponding $I_o$ values are between zero and maximum. Thus, the spatial distribution of the output light intensities is modulated by the spatial distribution written into the optical image, realizing the non-coherence/coherence transformation of images.

3. Other types of spatial light modulators

(1) The acousto-optic spatial light modulator

The acousto-optic spatial light modulator is a device for performing optical modulation using the acousto-optic effect. The operation of the acousto-optic modulator (whose structure is described in section 1.3 of this chapter) begins by transforming the electric write in signal into an ultrasonic wave carrying write in information via an electro-acoustic transducer. This acoustic wave acts on an acousto-optic medium to generate the distribution of an internal stress field which, in turn, is converted into the distribution of variation of the medium’s refractive index via the opto-elastic effect to constitute a “phase grating”. When passing through it, the read out light is modulated under the action of this “grating”. It is known from the previously discussed principle of the interaction between acoustic and light that the intensity of its diffracted light can be controlled by the power of the ultrasonic wave or, otherwise, by the electric driving power of the electro-acoustic transducer. Therefore, by changing the ultrasonic power, it is possible to attain light intensity modulation. If the frequency modulating function of the acousto-optic device is utilized, it is also possible to realize the phase modulation for the read out light. This is because the rate of variation of the light wave phase with time is proportional to the angular frequency $\omega$. Hence for light waves of different frequencies, after their propagation for identical periods of time, the amount of change in the phase is different.

However, compared with the previously described spatial light modulator, the acousto-optic spatial light modulator is different in two aspects; one is the spatial distribution of the write in information is not fixed but is slowly moving with the velocity of sound; second, the write in information is distributed only along the one-dimensional space (parallel to the propagation direction of the sound wave). So the acousto-optic modulator is most suitable for optically parallel processing of one-dimensional images (or information).

For instance, the acousto-optic modulator can be used to perform real-time frequency spectrum analysis of the broad frequency band radio frequency signal.

In radio astronomy, the composition of a celestial body can be learned by an analysis of the radio frequency electromagnetic wave radiated by it. A device for implementing frequency spectrum using the acousto-optic modulator is as shown in Fig. 1.6-6. The device is made up of two parts. The first part is the input circuit, including the receiving antenna A, the local oscillator (LO), the mixer M, and the power amplifier AMP. Following the mixing of the radio frequency signal RF with the local oscillation signal, the frequency falls from the radio frequency region to the ultrasonic region, the amplified ultrasonic signal being used to drive the electro-acoustic transducer T. The second part of the device is an integrated
optical device D, which carries out Ti diffusion on the surface of the LiNbO$_3$ substrate to form a waveguide and the transducer T. The acoustic wave excited by the transducer in this waveguide forms the acousto-optic grating G. In the waveguide layer, the two lenses $L_1$ and $L_2$ are also fabricated. On one side of the waveguide is a laser diode LD and on the other there’s an opto-electric detector DA.

![Figure 1.6-6 The acousto-optic frequency spectrum analyzing device](image)

The operation principle of the said frequency spectrum analyzing device is: the light beam emitted from LD, following collimation by $L_1$, serves as the incident light (read out light). This light beam experiences Bragg diffraction on the grating containing multiple frequency components while generating diffracted light beams in multiple directions at the same time. The intensity of all diffracted light beams depends on the average power of all the frequency components. After the detector array DA has detected the positions and intensities of the beams of diffracted light, it will be possible to find the frequency components and their relative intensities of the detected signals, completing the frequency spectrum analysis of the signals.

(2) The magneto-optic spatial light modulator

The magneto-optic spatial light modulator realizes modulation of the read out light using the magneto-optic effect after the write in information is recorded by inducing and magnetizing the ferromagnetic material.

1) The recording of the write in information. We know that some magnetic materials are magnetized when induced by an external magnetic field. When the external magnetic field is removed, the magnetic induction intensity is not restored as zero, but still has a “residual magnetic intensity”. Now, even if there is an external magnetic field in the opposite direction, as long as its intensity does not exceed the critical value, the direction of the above-mentioned residual magnetic intensity will still not change. Only after the size of the external magnetic field in the opposite direction exceeds the critical value, will the direction of the residual magnetic intensity change with it. Therefore, the direction of the magnetic material’s stable residual magnetic intensity can be utilized to “memorize” the direction of the original external magnetic field. If it is required to vary, then it is necessary to apply a sufficiently large magnetic field in the opposite direction. As there are two stable residual magnetic directions, the information recorded is binary. If a magnetic material is made into a thin film in shape, and divided into a large quantity of image elements independent of each other (etched into rectangular image elements arrays), by fabricating orthogonal addressing electrodes between one image element and the other, a two-dimensional data array represented with binary digits can be recorded.

The method of specific data recording is what is known as the matrix addressing, by which an electric current is applied on the electrode to generate a rather strong local magnetic field in the opposite direction at a certain unit that needs to change its residual magnetic
direction to the effect that the designated image element will have its residual magnetic
direction reversed. When the electric current flows through two electrodes in the orthogonal
direction, the image element at the junction of the electrodes will be addressed (which one of
the four image elements around the junction is addressed after all depending on the design
of the magneto-optic thin film and the direction of the current in the electrodes). The state
of magnetization of the thin film varies with the addressing magnetic field. Thus, by means
of line by line write in, the binary electric write in signals can be transformed into an array
of information as a 2-dimensional one characterized by the residual magnetic direction.

2) The information read out. For the magneto-optic modulator, the modulation of the
read out light is realized through the magneto-optic effect; that is, when a beam of linearly
polarized light passes the magneto optic medium, if there exists a magnetic field along the
direction of light propagation, then, because of the Faraday effect, the polarizing direction
of the incident light will rotate with the propagation of the light, the direction of rotation
depending on that of the magnetic field. Thus, we shall be able to transform the information
on the residual magnetic distribution recorded in the above-mentioned magnetic thin film
into a different distribution of the polarizing state of the output light. If an analyzer is also
used, then the binary amplitude modulation or phase modulation can be accomplished.

The specific modulating process can be illustrated with Fig.1.6-7. If the two image el-
ements “1” and “2” have already been modulated into residual magnetic intensity of the
opposite direction (denoted by direction of an arrow in the figure, in which “1” represents
that thin film magnetization has the same direction with the light beam, “2” the converse),
because of the Faraday effect, after the linearly polarized light polarizing along the y-axis
direction has passed through the two units, the polarizing direction will rotate a $\theta$ and $-\theta$
angle, respectively, to yield two streams of outgoing light $P_1$ and $P_2$, (one rotating a $\theta$ angle
clockwise, one rotating a $\theta$ angle counterclockwise). Then, have an analyzer $A$ arranged in
the rear of the device, whose light transmittance direction forms an angle $\varphi$ with the y-axis.
After $P_1$ passes through $A$, the light intensity is proportional to $\cos^2(\varphi - \theta)$ while after $P_2$
passes through $A$, the light intensity is proportional to $\cos^2(\varphi + \theta)$, realizing binary am-
plitude modulation. If the $\varphi$ angle is appropriately chosen so that $\varphi - \theta = \pm 90^\circ$, complete
contrast output can be obtained; that is, one image element is in the “closed state” with no
light passing through it while the other image element can pass through in part or in whole,
or in the “open state”. The magneto-optic spatial light modulator can reach a very high
frame rate and has a stable characteristic of storage as well as a very high contrast ratio. It
can be made into large array devices (e.g., those of 512 $\times$ 512 image elements). This kind
of spatial light modulator has found wide application in optical pattern recognition, optical
information processing, image coding, optical interconnection, etc.

![Fig. 1.6-7 The information readout of the magneto-optic modulator](image-url)
optic, and magneto-optic effect. Apart from these, there have emerged in recent years many other kinds of spatial light modulators such as ferroelectric ceramic (PLZI) modulators, microchannel plate (MSLM) modulators, multiquantum well modulators, etc., which will not be dealt with here.

**Exercises and questions for consideration**

1. A longitudinally applied KDP electro-optic modulator length is 2 cm and refractive index \( n = 1.5 \). If the operation frequency is 1 GHz (1000 kHz), try to find the transition time of light at this moment and the decay factor induced.

2. In order to obtain linear modulation from an electro-optic modulator, insert a 1/4 wave plate in the modulator. How would one best place its axial layer? If the 1/4 wave plate is rotated, what is the change in the dc bias it provides?

3. In order to reduce the half-wave voltage of the electro-optic modulator, adopt four pieces of \( z \)-cut KD*P crystal to connect (the optical path in series, the electric path in parallel) as a longitudinal series structure. (1) In order to make the electro-optic effect of the four pieces of crystal superimposed piece by piece, how should the \( x \)-, \( y \)-axes of the pieces of crystal be orientated? (2) If \( \lambda = 0.628 \) \( \mu \text{m} \), \( n_0 = 1.51 \), \( \gamma_63 = 23.6 \times 10^{-12} \text{ m/V} \), calculate its half-wave voltage and compare with the single crystals.

4. Try to design an experimental device, and explain how to examine the polarized state of the incident light (the linearly polarized light, the elliptically polarized light, and spontaneous light), and point out the phenomenon based on which your conclusion is made. If a longitudinal electro-optic modulator does not have a polarizer, can the incident spontaneous light be modulated? Why?

5. Suppose a lead molybdate (PbMoO\(_4\)) acousto-optic modulator is used to modulate the He-Ne laser. It’s known that the acoustic power \( P_a = 1 \) W, the acousto-optic interaction length \( L = 1.8 \) mm, the transducer width \( H = 0.8 \) mm, \( M_2 = 36.3 \times 10^{-15} \text{ s}^3/\text{kg} \). Find the Bragg diffraction efficiency of the lead molybdate acousto-optic modulator.

6. What reaction will a standing wave ultrasonic field make to the Bragg diffracted light? Give the frequency shift it brings about and the direction of diffraction.

7. Make an acousto-optic deflector using a lead molybdate crystal. Take \( n = 2.48 \), \( M_2 = 25 \) (relative to fused quartz \( M_{2\text{quartz}} = 1.51 \times 10^{-15} \text{ s}^3/\text{kg} \) ). The transducer length \( L = 1 \) cm, the width \( H = 0.5 \) cm. The acoustic wave propagates along the direction of the optical axis, the acoustic frequency \( f_0 = 150 \) MHz, \( \nu_s = 3.66 \times 10^5 \text{ cm/s} \), the light beam width \( w = 0.85 \) cm, \( \lambda = 0.5 \mu \text{m} \). (1) Prove the said deflector can only generate normal Bragg diffraction. (2) To obtain 100% diffraction efficiency, find the sound power \( P_s \). (3) If we take the Bragg bandwidth \( \Delta f = 125 \) MHz, how much is the diffraction efficiency reduced? (4) Find the resolvable number of points \( N \).

8. A beam of linearly polarized light passes through a solid glass rod of \( L = 25 \) cm, diameter \( D = 1 \) cm, its exterior wound by \( N = 250 \) coils of conductor, energized with current \( I = 5 \) A. Take Verdet constant \( V = 0.5^\prime/\text{G} \cdot \text{cm} \). Try to calculate the rotation angle \( \theta \) of light.

9. Give an account of the difference and association between the waveguide modulator and volume modulator using electro-optic modulation as an example.

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CHAPTER 6. The Frequency Stabilizing Technology


CHAPTER 7. The Nonlinear Optical Technology


CHAPTER 8. The Laser Transmission Technology


